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Вестник Института математики НАН КР

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The journal is focused on the publication of scientific articles in all areas of mathematics and computer science. It is published in English, twice a year, in print and electronic form. All articles are subject to peer review. Published articles are in the public domain. The journal is included into the List of editions for publishing main scientific results of dissertations, by the Resolution of Presidium of the Highest Attestation Commission of the Kyrgyz Republic No. 142, December 29, 2020.

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BORUBAEV Altai Asylkanovich

A cademician, doctor of physical and mathematical sciences, professor

In 2022, Academician A.A. Borubaev made a scientific discovery called "The permutability property of operations of the absolute, replenishment and expansion of a single space", confirmed by the International Academy of Authors of Scientific Discoveries and Inventions (09/23/2022, Diploma No. 74-C).). For a discovery in the field of mathematics, Academician A.A. Borubaev was awarded the Silver Medal of Honor. P. Kapitsa (Resolution of the Presidium of the Russian Academy of Natural Sciences No. 316 of 09/06/2022).

The essence of scientific discovery is as follows:

1) the previously unknown basic properties of the commutativity of the operation of the absolute and the most important extensions of the uniform space are established,

2) important organic connections are found between the key concepts of the theory of homogeneous and topological spaces.

On October 28, 2022, the Institute of Mathematics, together with the Kyrgyz Mathematical Society, held the International Scientific Conference "IV Borubaev Readings" at the National Academy of Sciences of the Kyrgyz Republic. Within the framework of the conference, a solemn presentation to Academician A. A. Borubaev Diploma and Silver Medal. P. Kapitsa for a new scientific discovery in the field of mathematics.

At the plenary session, a report was made by: Sadovnichiy Yu.V., Doctor of Physical and Mathematical Sciences, Professor, Head of the Department of General Topology and Geometry, Faculty of Mechanics and Mathematics, Moscow State University named after M.V. Lomonosov (Russia); Lyubisha D.R. Kocinac, Doctor of Physical and Mathematical Sciences, Professor, Honorary Professor of the Faculty of Natural Sciences and Mathematics of the University of Nis (Serbia); Zaitov A. A., Doctor of Physical and Mathematical Sciences, Professor, Head of the Department of Mathematics and Natural Disciplines of the Tashkent Institute of Architecture and Civil Engineering (Uzbekistan); Sopuev A., Doctor of Physical and Mathematical Sciences, Professor, Osh State University, etc.

Within the framework of the conference, the presentation of the monograph by A.A. Borubaev "Unified topology and its applications", containing a scientific discovery.

The conference discussed the development of methods for studying modern problems in the field of topology, geometry and functional analysis, differential and integral equations and methods for optimizing complex systems.

Issue No. 2, 2022 of the publication “Bulletin of the Institute of Mathematics of the National Academy of Sciences of the Kyrgyz Republic”, presents the materials of the plenary reports of the International Scientific Conference “IV Borubaev Readings”.

ДИПЛОМ

НА ОСНОВАНИИ
РЕШЕНИЯ ПРЕЗИДИУМА
РОССИЙСКОЙ АКАДЕМИИ
ЕСТЕСТВЕННЫХ НАУК
№316 ОТ 06.09.2022

за открытие
в области математики
(диплом №74-S)

РАЕН

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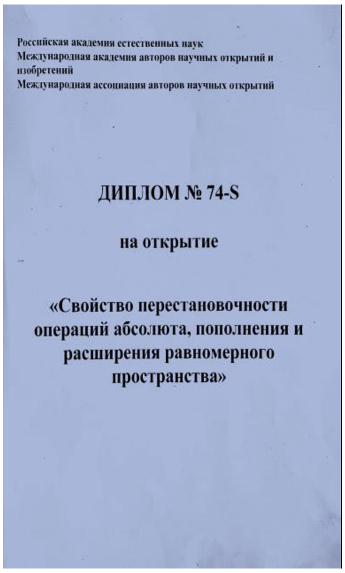
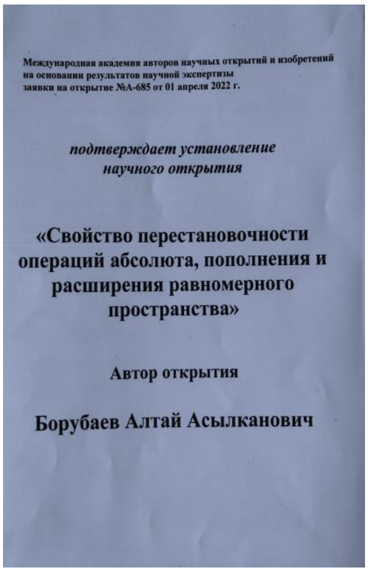


Российская академия естественных наук
Международная академия авторов научных открытий и изобретений
Международная ассоциация авторов научных открытий

ДИПЛОМ № 74-S

на открытие

«Свойство перестановочности операций абсолюта, пополнения и расширения равномерного пространства»



Международная академия авторов научных открытий и изобретений
на основании результатов научной экспертизы
заявки на открытие №А-685 от 01 апреля 2022 г.

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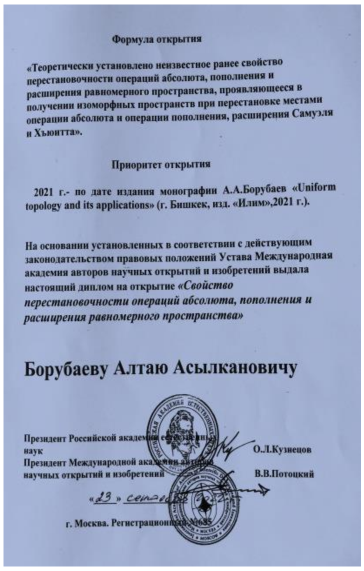
«Свойство перестановочности операций абсолюта, пополнения и расширения равномерного пространства»

Автор открытия

Борубаев Алтай Асылканович



Формула открытия

«Теоретически установлено неизвестное ранее свойство перестановочности операций абсолюта, пополнения и расширения равномерного пространства, проявляющееся в получении изоморфных пространств при перестановке местами операции абсолюта и операции пополнения, расширения Самуэля и Хьюитта».

Приоритет открытия

2021 г.- по дате издания монографии А.А.Борубаев «Uniform topology and its applications» (г. Бишкек, изд. «Илим»,2021 г.).

На основании установленных в соответствии с действующим законодательством правовых положений Устава Международная академия авторов научных открытий и изобретений выдала настоящий диплом на открытие *«Свойство перестановочности операций абсолюта, пополнения и расширения равномерного пространства»*

Борубаеву Алтаю Асылкановичу

Президент Российской академии естественных наук — О.Л.Кузнецов
Президент Международной академии авторов научных открытий и изобретений — В.В.Потоцкий

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ON SOME PROPERTIES OF THE SUPEREXTENSION FUNCTOR

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In this paper we study certain cardinal and homotopy properties of the space of maximal linked systems. It is proved: (1) The space λ_3^0S of the Sorgenfrey line S contains a closed discrete subset of cardinality c; (2) Let A be a subset of R and B ⊆ R \ A, ϕ be either the cardinal invariant d, e, c (resp. wd, l, πw). If B is a subset of the Hattori space (R, τ (A)) which is homeomorphic to the Sorgenfrey line S, then hϕ(λ_3^0(R, τ (A))) = c (resp. hϕ(λ_3^0(R, τ (A))) ≥ c), where hϕ is the hereditary invariant of ϕ; (3) If the mappings f, g: X → Y are homotopic, then the mappings λf, λg : λX → λY are also homotopic.

Keywords: Density, weak density, spread, maximal linked system, functor, fundamental group, retract.

Бул макалада биз максималдуу байланышкан системалардын мейкиндигинин кээ бир кардиналдык жана гомотопиялык касиеттерин изилдейбиз. Далилденди: (1) Соргенфри сызыгынын S λ_3^0S мейкиндиги c кардиналдуулуктун туюк дискреттик чакан жыйындысын камтыйт;(2) A R жана B ⊆ R \ A кичи жыйындысы болсун, ϕ же d, e, cнын кардиналдык инварианты болсун (тиешелүү түрдө wd, l, πw). Эгерде B Хаттори мейкиндигинин чакан жыйындысы (R, τ (A)), Соргенфри сызыгына S гомеоморфтук болсо, анда hϕ(λ_3^0 (R, τ (A)))) = c (тиешелүүлүгүнө жараша, hϕ(λ_3^0) (R, τ (A))) ≥ c), мында hϕ - ϕнун тукум куума инварианты; (3) Эгерде f, g : X → Y сүрөттөрү гомотоптук болсо, анда λf, λg : λX → λY сүрөттөрү да гомотоптук болуп саналат.

Урунтуу сөздөр: Тыгыздык, начар тыгыздык, таралуу, максималдуу байланышкан система, функтор, фундаменталдык топ, чегинүү.

В работе изучаются некоторые кардинальные и гомотопические свойства пространства максимальных сцепленных систем. Доказано: (1)пространство λ_3^0S прямой Зоргенфрея S содержит замкнутое дискретное подмножество мощности c; (2) Пусть A — подмножество R и B ⊆ R \ A, ϕ — либо кардинальный инвариант d, e, c (соответственно wd, l, πw). Если B — подмножество пространства Хаттори (R, τ (A)), гомеоморфное прямой Зоргенфрея S, то hϕ(λ_3^0(R, τ (A))) = c (соответственно hϕ(λ_3^0(R, τ (A))) ≥ c), где hϕ — наследственный инвариант ϕ; (3) Если отображения f, g:X → Y гомотопны, то гомотопны и отображения λf, λg : λX → λY.

Ключевые слова: Плотность, слабая плотность, спред, максимальная связанная система, функтор, фундаментальная группа, ретракт.

1. Introduction

The superextension λX of a topological space X was introduced by De Groot in [10]. Its construction is similar to the construction of Wallman compactifications. This topic attracted many researches, and they demonstrated that λX has many nice properties. This space λX is of great interest for researchers, since, among others, it contains the hyperspace expX. J. van Mill and M. van de Vel [27] showed that for a compact space X the Wallman topology on λX coincides with the topology induced by the natural imbedding $\lambda X \rightarrow \exp(\exp X)$.

Recall that a covariant functor is a mapping F which assigns to a topological space X the space F(X), and to a continuous mapping $f:X \rightarrow Y$, the mapping $F(f): F(X) \rightarrow F(Y)$ satisfying the following conditions:

(i) F preserves identity, that is, if idX is the identity mapping of X, then $F(idX) = id_{F(X)}$;

(ii) F preserves composition, that is, if $f:X \rightarrow Y$ and $g:Y \rightarrow Z$ are continuous mappings, then we have $F(g \circ f) = F(g) \circ F(f)$. We refer the reader to the book [9] for more information about functors, and the book [28] for the superextension. In [14], the superextension functor λ:Comp $\rightarrow$ Comp acting in the category of compact spaces and their continuous mappings was studied. It was proved that the functor λ: Comp $\rightarrow$ Comp satisfies the normality conditions, except the preimage preservation condition (see also [15, 16, 17, 18]). In [23], topological and cardinal properties of the superextension of a topological space X were investigated. It was proved that the functor λ: Comp $\rightarrow$ Comp preserves the topological property admissible extensions. In [24], the authors discussed some cardinal invariants (density, weak density, π−weight, Shanin number, cellularity, caliber and pre-caliber) for the superextensions of Hattori spaces. It was proved in [30] that the caliber of the superextension of a topological space X is caliber for the N-subtle kernel of X. For related investigations in recent years see, for example, [2, 3, 5, 6, 7, 19, 20, 25, 29, 30]. The current paper is devoted to the investigation of (hereditary) cardinal invariants such as the hereditary density, hereditary weak density, and hereditary Lindel¨of number. Furthermore, the relation between spread and extent of the space λ_3^0 (R, τ (A)) of the superextension of the Hattori space is studied. Moreover, it is shown that the functor λ preserves the homotopy and the retraction of topological spaces. As a consequence, we prove that the functor λ is a covariant homotopy functor. More precisely, the paper is organized as follows. In Section 2, we recall basic notions and notations that will be used in the rest of the study. In Section 3, we study hereditary cardinal invariants and results for the space λ_3^0(R, τ (A)) of the superextension of the Hattori space. Finally, in Section 4, we study some geometric properties for the space λX of the superextension of a topological spaces X. Throughout the paper κ denotes an infinite cardinal number, and ω and c denote the countable cardinal number and continuum. By Comp we denote the category of compact spaces and their continuous mappings.

2. Preliminaries

In this section we give some notations, concepts, and statements that are widely used in this article.

A system $\xi = \{F_\alpha: \alpha \in A\}$ of closed subsets of a space X is called *linked* if every two elements of ξ have non-empty intersection. Each linked system can be filled up to a maximal linked system, but such a completion is not unique.

The set of all maximal linked systems of a space X we denote by λX.

For an open set $U \subset X$ we set.

$$O(U) = \{\xi \in \lambda X : \text{there exists } F \in \xi \text{ such that } F \subset U\}.$$

The family of all sets of the form O(U) covers the set λX, hence it is a subbase of a topology on λX. The set λX with this topology, is called the *superextension* of X. The family of all sets of the form $O(U_1, U_2, ..., U_n)=\{\xi \in \lambda X: \text{for } i = 1, 2, ..., n \text{ there exist } F_i \in \xi \text{ such that } F_i \subset U_i\}$, where $U_1, U_2, ..., U_n$ are open sets in X, forms a base of the topology on λX.

Note 2.1 Similarly, for a closed set $C \subset X$ we set $C^+ = \{\xi \in \lambda X: C \in \xi\}$.

The family of sets of the form C^+, C closed in X, is a closed subbase in the space λX. Let $\xi \in \lambda X$. The *support* of ξ, denoted by supp(ξ), is defined as the intersection of all closed subsets $Y \subset X$ for which $\xi \in \lambda(iY)(\lambda(Y))$, where $iY: Y \to X$ is the inclusion.

We consider also functors of finite degrees obtained by taking subfunctors λ_n of the functor λ, where $\lambda_n X = \{\xi \in \lambda X : |supp(\xi)| \leq n\}$.

We use the following notation: $\lambda_3^0 X = \lambda_3 X \setminus \lambda_2 X$.

Let us recall (cf. [8]) that the Sorgenfrey line $S = (R, \tau S)$ is the set R of real numbers with the lower limit topology τS (the family of all half-open intervals $[a, b)$, where $a < b$, is a base of the topology τS). In [11], Hattori introduced a family $G = \{\tau(A): A \subseteq R\}$ of topologies on R such that $\tau(R) = \tau_E$ (the Euclidean topology on R) and $\tau(\emptyset) = \tau_S$. The topology $\tau(A)$ on R is defined as follows:

(1) For each $x \in A$, $\{(x - \varepsilon, x + \varepsilon) : \varepsilon > 0\}$ is the neighborhood base at x, and

(2) For each $x \in R \setminus A$, $\{[x, x + \varepsilon): \varepsilon > 0\}$ is the neighborhood base at x. It is easy to see that for any $A, B \subseteq R$ we have $A \supseteq B$ if and only if $\tau(A) \subseteq \tau(B)$. In particular, $\tau_E \subseteq \tau(A) \subseteq \tau_S$ for each $A \subseteq R$. The spaces $(R,\tau(A)), A \subset R$, possess several nice topological properties (see [3,4]). In this note we will call the spaces $(R,\tau(A))$, $A \subset R$, the Hattori spaces. For a space X we denote by *d(X), c(X), L(X), s(X), e(X)* the density, cellularity, Lindel¨of number, spread, and extent of X, respectively [1, 8, 12]. A collection of nonempty open sets B is said to be a π-base of X if for every nonempty open set G there is a $B \in B$ with $B \subset G$. The π-weight of a space X, denoted by $\pi w(X)$ is defined as $\pi w(X) = \min\{|B| : B \text{ is a } \pi \text{ base of } X\}$ [8]. We say that the weak density of a space X, denoted by wd(X), is $\tau \geq \omega$, if τ is the smallest cardinal number such that there exists a π-base in X which is the union of τ many centered systems of open sets [2].Let ϕ be a cardinal invariant. Put $h\phi(X) = \sup\{\phi(Y) : Y \subseteq X\}$.The cardinal invariant $h\phi$ is called a hereditary cardinal invariant of ϕ. It is evident that $\phi(X) \leq h\phi(X)$ for any topological space X [8].

3. Some cardinal properties of the functor λ. It is well known that the set $D = \{(x, x, -x): x \in R\}$ of the space S^3 is closed and discrete, hence $s(S^3) = e(S^3) = c$. It is also easy to see that $\pi w(S^3) = c(S^3 = \omega$. We will generalize these facts.

Lemma 3.1. *The space λ 0 3S contains a closed discrete subset of cardinality c.*

Proof. Note that the subset $Y = \{(x, x, z) \in S^3 : x \geq z\}$ of S^3 is homeomorphic to the space $\lambda_3^0 S$, and the set $Z = \{(x, x, z) \in Y : z = -x, x > 0\}$ is closed and discrete in Y and has cardinality c. ¤

Let Y be a subset of a topological Hausdorff space X and $\mu Y = \{Z : Z = \{F \in \mu \in \lambda_3^0 X: F \subset Y\}\}$. Denoted by $\lambda_\mu X$ the family of all systems μY for all $Y \subset X$. It is easy to see that $\lambda_\mu X \subset \lambda_3^0 X$. The following statement is true.

Lemma 3.2. Let Y be a subset of a topological Hausdorff space X and $\lambda_\mu X$ the family of all systems μY for $Y \subset X$. Then:

(i) the space $\lambda_3^0 Y$ is homeomorphic to the subspace $\lambda_\mu X$ of the space $\lambda_3^0 X$,

(ii) the set $\lambda_\mu X$ is open (closed,clopen) in $\lambda_3^0 X$ whenever Y is open(closed,clopen) in X.

Proof.(i) It is known [22] (and easy to check) that the space $\lambda_3^0 Y$ is homeomorphic to the subspace $\lambda_\mu X$ of the space $\lambda_3^0 X$. In [22], it is shown that the space $\lambda_3^0 Y$ is homeomorphic to the space exp $exp_3^0 Y$. Hence, we have that the space $\lambda_3^0 Y$ is homeomorphic to $\lambda_\mu X \subset \lambda_3^0 X$.

(ii) follows from the fact that the set $\lambda_\mu X$ is open (closed, clopen) in $\lambda_3^0 X$ whenever Y is open (closed, clopen) in X [22] and the mentioned result that $\lambda_3^0 X$ is homeomorphic to $exp_3^0 X$. ¤

From Lemmas 3.1 and 3.2 we have the following.

Proposition 3.1. Let A be a subset of R and $B \subseteq R\backslash A$. If B is a (resp. closed) subset of (R, τ (A)) which is homeomorphic to S, then the space λ_3^0(R, τ (A)) contains a (resp. closed) discrete subset of cardinality c. Proof. In [4], it was shown that if $A \subseteq R$ and $B \subseteq R \setminus A$, then $\tau (A) |B= \tau S |B$. Recall that the set (R, τ (A)) is a closed subset of λ_3^0 (R, τ (A)). Hence, each (closed) discrete subset M of (R, τ (A)) with cardinality c (which exists by Lemma 3.1) is a (closed) discrete subset of λ_3^0 (R, τ (A)) with cardinality c. ¤

Proposition 3.2. Let A be a subset of R, and Y be a subspace of (R, τ (A)). Then $|\tau (A)|Y| \leq$ c. In addition, $|\lambda_3^0 Y| \leq$ c.

Proof. It is enough to show that $|\tau (A)| \leq$ c. Let B be a base for (R, τ (A)) of cardinality $\leq$ c. Since the space (R, τ (A)) is hereditary Lindel¨of, each open subset of (R, τ (A)) is an union of countably many elements of B. Hence, $|\tau (A)| \leq c\ \omega = (2\omega)\ \omega = 2\omega =$ c. ¤

Corollary 3.1. Let A be a subset of R and B ⊆ R\A. If B is a closed subset of (R, τ (A)) which is homeomorphic to S, then s(λ_3^0(R,τ (A)))=e(λ_3^0(R,τ (A))) = c.

Proof. By Proposition 3.1 we have e(λ_3^0(R, τ (A))) $\geq$ c.

Note that

$$e(\lambda_3^0(R, \tau (A))) \leq s(\lambda_3^0(R, \tau (A))) \leq |(\lambda_3^0\ (R, \tau (A))|.$$

It follows from Proposition 3.2 that

$$|(\lambda_3^0(R, \tau (A)))| \leq |\exp(R, \tau (A))| \leq c.$$

Thus we get

$$s(\lambda_3^0\ (R, \tau (A))) = e(\lambda_3^0(R, \tau (A))) = c$$

which proves the corollary. ¤

Corollary 3.2. Let A be a subset of R and B ⊆ R \ A. If B is a subset of (R, τ (A)) which is homeomorphic to the space S, then s(λ_3^0(R, τ (A))) = c.

Proof. By Proposition 3.1 we have s(λ_3^0(R, τ (A))) $\geq$ c.

Note that s(λ_3^0(R, τ (A))) $\leq$ |(λ_3^0(R, τ (A)))|.

It follows from Proposition 3.2 that |(λ_3^0(R, τ (A)))| $\leq$ c.

Thus we get s(λ_3^0(R, τ (A))) = c. ¤

Corollary 3.3. Let A be a subset of R and B⊆R\A, such that B is a subset of (R, τ (A)) homeomorphic to (S. If ϕ is either the cardinal invariant d, e, c (resp. wd,L,πw), then hϕ(λ_3^0(R,τ(A))) = c (resp. hϕ(λ_3^0(R, τ(A))) $\geq$ c). **Proof.** In fact, note that he = s and hc = s. Since hc(λ_3^0(R, τ (A))) $\geq$ c (by Proposition 3.1), we have the equalities. The inequalities also trivially follow from Proposition 3.1.

4. Some geometric properties of the functor λ

In this section we study some geometric properties of the space of maximal linked systems. We begin with definitions of notions that will be used in this section. We mainly follow terminology from [21] and [26]. Continuous mappings f, g: $X \to Y$ are said to be homotopic if there is a continuous mapping H: $X \times I \to Y$ such that H(x, 0) = f(x) and H(x, 1) = g(x). The mapping H is called a homotopy between f and g and we write f ' g [21]. A continuous mapping f: $X \to Y$ is said to be a homotopy equivalence if there exists a continuous mapping g: $Y \to X$ such that the compositions $g \circ f$ and $f \circ g$ are homotopic to the identity mappings on X and Y respectively. Two topological spaces X and Y is said to be homotopically equivalent (notation, X ' Y) if there exists a homotopy equivalence f: $X \to Y$ [21].

Example 4.1. An example of a pair of homotopy equivalent spaces is provided by the cylinder C and the circle S 1. To see this write C and S 1 as

$C = \{(x, y, z) \in R^3: x^2 + y^2 = 1, -1 \le z \le 1\}$,

$S^1 = \{(x, y, z) \in R^3: x^2 + y^2 = 1, z = 0\}$.

Define i: S 1 → C as an inclusion mapping and $r : C \to S^1$ by r(x, y, z) = (x, y, 0). Obviously $ri = id_S{}^1 : S^1 \to S^1$ whereas H: $C \times I \to C$ defined by H((x, y, z), t) = (x, y, tz) is a homotopy between ir and id_C: $C \to C$.

By a *covariant homotopy functor* we mean a functor F in the category **Top** of topological spaces and their continuous mappings satisfying. (*) F preserves homotopy, that is, if a mapping H(x, t) is a homotopy between continuous mappings f, g: $X \to Y$, then F(H(x, t)) is also a homotopy between mappings F(f), F(g): $F(X) \to F(Y)$. A continuous mapping f: $[0, 1] \to X$ is called a *path* in X. The point f(0) is called the initial point and f(1) is called the final or terminal point of this path. If $x \in X$, then one defines e_x: $I \to X$ as the constant path, i.e. $e_x(t) = x$ for any $t \in I$. A topological space X is said to be *path-connected* if given any two points x_0, x_1 in X there is a path in X from x_0 to x_1. A space X is said to be *contractible* if it is homotopy equivalent to a point. A subset A of a topological space X is called *a retract* of X if there exists a continuous mapping r: X → A such that r ¹ A = id_A. The mapping r is called a retraction [21].

A property P of topological spaces is called a *homotopy property* if it is preserved by all homotopy equivalences. Precisely, P is a homotopy property if and only if for an arbitrary homotopy equivalence f : X → Y , X has P implies that Y also has P.

Theorem 4.1. *If mappings f, g: X → Y are homotopic, then the mappings λf, λg: λX → λY are also homotopic.*

Proof. Assume that the mappings f, g: X → Y are homotopic. Then there exists a continuous mapping H: X × I → Y such that H(x, 0) = f(x), and H(x, 1) = g(x). On the other hand, we have that λf(ξ) = {f(L) : L ∈ ξ}, λg(ξ) = {g(L) : L ∈ ξ}. Now we can define the mapping

λH(ξ, t) = {H(L, t) : L ∈ ξ, t ∈ [0, 1]}, where H(L, t) = {H(x, t) : x ∈ L, t ∈ [0, 1]}.

It is clear that since the mapping H is continuous, the mapping λH is also continuous. Now we will show that the mapping λH is a homotopy between the mappings λf and λg.

Indeed, λH(ξ, 0) = {H(L, 0) : L ∈ ξ} = {f(L): L ∈ ξ} = λf(ξ); λH(ξ, 1) = {H(L, 1): L ∈ ξ} = {g(L): L ∈ ξ} = λg(ξ). This means that λf ≃ λg. ¤

Corollary4.1. *If spaces X and Y are homotopically equivalent, then the spaces λX and λY are also homotopically equivalent.*

Proof. Suppose that the spaces X and Y are homotopically equivalent. Then there exist two continuous mappings f : X → Y and g : Y → X such that f ∘ g ≃ idY and g ∘ f ≃ id_X. It means that there are two homotopy $H_1(y, t)$ and $H_2(x, t)$ such that $H_1(y, 0) = (f \circ g)(y)$, $H_1(y, 1) = y$ and $H_2(x, 0) = (g \circ f)(x)$, $H_2(x, 1) = x$.

We can define the compositions λf ∘ λg : λY → λY and λg ∘ λf : λX → λX of the mappings λf : λX → λY and λg : λY → λX as follows:

(λf ∘ λg)(η) = {(f ∘ g)(K): K ∈ η} and λg ∘ λf)(ξ) = {(g ∘ f)(L): L ∈ ξ}.

One can easily check that the mapping $\lambda H_1(\eta, t) = \{H_1(K, t) : K \in \eta, t \in I\}$ is a homotopy between λf ∘ λg and idλY . Indeed,

λH1(η, 0) = $\{H_1(K, 0) : K \in \eta\}$ = {(f ∘ g)(K) : K ∈ η} = (λf ∘ λg)(η) and $\lambda H_1(\eta, 1) = \{H_1(K, 1) : K \in \eta\} = \{K : K \in \eta\} = \eta = id_{\lambda Y}$.

Similarly, the mapping $\lambda H_2(\xi, t) = \{H_2(L, t) : L \in \xi, t \in I\}$ is a homotopy between $\lambda g \circ \lambda f$ and idλX. Indeed,

$$\lambda H_2(\xi, 0) = \{H_2(L, 0) : L \in \xi\} = \{(g \circ f)(L) : L \in \xi\} = (\lambda g \circ \lambda f)(\xi) \text{ and}$$

$$\lambda H_2(\xi, 1) = \{H_2(L, 1) : L \in \xi\} = \{L : L \in \xi\} = \xi = id_{\lambda X}.$$

It means that λX and λY are homotopically equivalent. ¤

Proposition 4.1. *If a set A is a retract of a topological space X, then the set λA is a retract of the space λX.*

Proof. Suppose that A is a retract of X. Then there exists a continuous mapping r: X → A such that r(a) = a for all a ∈ A. Now we consider the mapping λr: λX → λA. For every $\xi' \in \lambda A$ we have that $\lambda r(\xi') = \{r(L') : L' \in \xi'\}$, where $r(L') = \{r(a) : a \in L'\}$. So $r(L') = L'$, hence $\lambda r(\xi') = \xi'$. It means that the mapping λr : λX → λA is a retraction. Hence, the set λA is a retract of the space λX. ¤

Theorem 4.2. The functor λ is a covariant homotopy functor.

Proof. Now we will show that the functor λ satisfies the above three conditions.

(i) Let id_X be identity mapping in the topological space X. Then we have that $\lambda id_X(\xi) = \{id_X(L): L \in \xi\} = \{L: L \in \xi\} = \xi$. It means that the mapping λidX is an identity mapping in the topological space λX.

(ii) Let f : X → Y , g : Y → Z be the continuous mappings. Then it follows that $\lambda(g \circ f)(\xi) = \{(g \circ f)(L) : L \in \xi\} = \{g(f(L)) : L \in \xi\} = \lambda g(f(\xi)) = \lambda g(\xi) \circ \lambda f(\xi)$. (*) It follows easily from Theorem 4.1. ¤

In [13], some propositions about homotopy properties of topological spaces were given. For instance, contractibility is a homotopy property of the spaces. We have the following.

Corollary 4.2. If a topological space X is contractible, then the space λX is also contractible.

Proof. Assume that X is a contractible space. It means that X ' {a}. By Corollary 4.1 it implies immediately that $\lambda X \simeq \lambda\{a\}$, which means that λX is homotopy equivalent to the point λ{a}. ¤

If f and g are two paths in X with f(1) = g(0), then by the product of f and g we mean the path $f * g$, which is defined by

$$(f * g)(t) = \begin{cases} f(2t), \ if \ 0 \le t \le 1/2, \\ g(2t-1), \ if \ 1/2 \le t \le 1. \end{cases}$$

Let $f \simeq g$ and $g \simeq h$, and let H_1 be a homotopy from f to g and H_2 a homotopy H_2 from g to h. Then $f \simeq h$. Define a homotopy $H: X \times I \to Y$ between f and h as follows:

$$H(x, t) = \begin{cases} H_1(x, 2t), \ if \ 0 \le t \le 1/2, \\ H_2(x, 2t-1), \ if \ 1/2 \le t \le 1. \end{cases}$$

Corollary 4.3. If the mapping $f: I \to X$ is path in X from the point x_0 to the point x_1, then a mapping $\lambda f : I \to \lambda X$ defined by $\lambda f(t) = \{f(t) : t \in [0, 1]\}$, is a path from the point $\xi_0 = \{L_0 : f(0) \in L_0\}$ to the point $\xi_1 = \{L_1 : f(1) \in L_1\}$ in λX.

Corollary 4.4. Let λf and λg be paths from $\xi_0 = \{L_0 : f(0) \in L_0\}$ to $\xi_1 = \{L_1 : f(1) \in L_1\}$ and from $\eta_0 = \{M_0 : g(0) \in M_0\}$ to $\eta_1 = \{M_1 : g(1) \in M_1\}$, respectively, with $\lambda f(1) = \lambda g(0)$. Then we can define the multiplication of the paths in λX as follows: $(\lambda f * \lambda g)(t) = \{(f * g)(t) : t \in [0, 1]\}$.
This defines a path from the points $\xi_0 = \{L_0: f(0) \in L_0\}$ to the point $\eta_1 = \{M_1: g(1) \in M_1\}$.

In [21] it was shown that the multiplication of equivalence classes of paths is associative; $([f][g])[h] = [f]([g][h])$.

Let f and g be paths with the initial point x_0 and the end point x_1. If there exists a homotopy H from f to g such that for every $t \in I$, $H(0, t) = x_0$ and $H(1, t) = x_1$, then f and g are path-homotopic. Let f be a path. Let [f] denote the equivalence class of all paths that are path-homotopic to f.

The operation $*$ can be applied to homotopy classes as well. Once again, let $f:I \to X$ be a path from x0 to x1 and let $g: I \to X$ be a path from x_1 to x_2. Define $[f] * [g] = [f * g]$.

Let X be a space and let x_0 be a point of X. A path in X that begins and ends at x_0 is called a loop based at x_0. Let $\pi_1(X, x_0)$ denote the set of all equivalence classes [f] of loops in X based at x_0. $\pi_1(X, x_0)$, when paired with the operation $*$, is a group with the identity element $[e_x]$ and an inverse element is *[f(t)] = [f(1 − t)]* for every *[f]*. We call $\pi_1(X, x_0)$ the fundamental group.

Suppose $(G, *)$ and $(G_1, *_1)$ are groups. A homomorphism is a mapping such that $f(x * y) = f(x) *_1 f(y)$ for all $x, y \in G$.

The fundamental groups of a space and its quotient space are not always isomorphic.

Example 4.2. Let $X = [0, 1] \times [0, 1]$ and Y be the M¨obius band. Clearly, Y is the quotient space of X. It is known [26] that X has the trivial fundamental group, while the fundamental group of Y (which is homotopy equivalent to the circle [21]) is isomorphic to the group (Z, +).

Corollary 4.5. The fundamental groups $\pi_1(X, x_0)$ and $\pi_1(\lambda X, \xi x_0)$ of the topological spaces X and λX are not always isomorphic for every $x_0 \in X$, where $\xi x_0 = \{L x_0 : x_0 \in L x_0 \}$.

Suppose that $h: X \to Y$ is a continuous mapping that carries the point x_0 of X to the point y_0 of Y . We denote this fact by writing $h: (X, x_0) \to (Y, y_0)$. If f is a loop in X based at x_0, then the composition $h \circ f: I \to Y$ is a loop in Y based at y_0. The correspondence $f \to h \circ f$ thus gives rise to a mapping carrying $\pi_1(X, x_0)$ into $\pi_1(Y, y_0)$. Define $h*: \pi_1(X, x_0) \to \pi_1(Y, y_0)$ by the equation $h*([f]) = [h \circ f]$. In [26] it was proved that $h*$ is a homomorphism. Now we will consider a mapping $\lambda h: X \to \lambda X$ which is defined by $\lambda h(x) = \{h(x): x \in X\}$.

Corollary 4.6. The fundamental groups $\pi_1(X, x_0)$ and $\pi_1(\lambda X, \xi x_0)$ of the topological spaces X and λX are homomorphic for every $x0 \in X$, and the homomorphism is defined by $(\lambda h)*[f] = [\lambda h \circ f]$, where $\xi x_0 = \{Lx0 : x0 \in Lx_0 \}$.

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BOUNDARY VALUE PROBLEMS FOR A MIXED PARABOLIC-HYPERBOLIC EQUATION OF THE THIRD ORDER WITH DISCONTINUOUS GLUING CONDITIONS

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The existence and uniqueness of the solution of the boundary value problem for the equation of the mixed parabolic-hyperbolic type of the third order with constant coefficients at the lower terms is proved. The solution of the problem by lowering the order for an equation of hyperbolic type is reduced to the Gursa problem and for an equation of parabolic type - to the first boundary value problem. The solvability of the problem is reduced to the solvability of the Fredholm integral equation of the second kind. After determining the trace of the function and its derivatives y of the first and second orders, the solution of the problem is completely determined in the areas under consideration.

Keywords: boundary value problems, existence, uniqueness, Green's function, Riemann's function, integral equations, reduction method.

Үчүнчү тартиптеги турактуу кичине мүчөлөрү бар аралаш параболикалык-гиперболикалык теңдеме үчүн чек аралык маселенин чечиминин жашашы жана жалгыздыгы дадилденген. Тартибин төмөндөтүү методу менен маселени чечүү гиперболикалык типтеги теңдеме үчүн Гурстун маселесини жана параболикалык типтеги теңдеме үчүн биринчи чек аралык меселеге келтирилет. Маселенин чечилиши Фредгольмдун экинчи түрдөгү интегралдык теңдемесинин чечилүүчүлүгүнө алып келинет. Изделүүчү функциянын жана анын у боюнча биринчи жана экинчи тартиптеги туундуларынын издери табылгандан кийин маселенин каралып жаткан областтардагы чечимдери толугу менен аныкталат.

Ачкыч сөздөр: чек аралык маселелер, чечимдин жашашы, чечимдин жалгыздыгы, Гриндин функциясы, Римандын функциясы, интегралдык теңдеме, тартибин төмөндөтүү методу.

Доказана существование и единственность решения краевой задачи для уравнения смешанного параболо-гиперболического типа третьего порядка с постоянными

коэффициентами при младших членах. Решение задачи методом понижения порядка для уравнения гиперболического типа сводится к задаче Гурса и для уравнения параболического типа - к первой краевой задаче. Разрешимость задачи сводится к разрешимости интегрального уравнения Фредгольма второго рода. После определения следа функции и её производных по y первого и второго порядков, решение задачи полностью определяется в рассматриваемых областях.

Ключевые слова: краевые задачи, существование, единственность, функция Грина, функция Римана, интегральное уравнение, метод понижения порядка.

1. Problem statement. In a rectangle $D=\{(x,y): 0<x<\ell, -h_1<y<h\}$ $(h,\, h_1>0)$ consider the equation

$$L_1L_2u=0, \tag{1}$$

where $L_1 \equiv \begin{cases} \dfrac{\partial^2}{\partial x^2}-\dfrac{\partial}{\partial y}+c_1, & y>0, \\ \dfrac{\partial^2}{\partial x \partial y}+c_2, & y<0, \end{cases}$ $\quad L_2 \equiv \dfrac{\partial}{\partial y}$, $c_1,\, c_2$ – given constants.

Let $D_1=D\cap(y>0)$, $D_2=D\cap(y<0)$. Note that the equation

$$L_1L_2u \equiv \left(\frac{\partial^2}{\partial x^2}-\frac{\partial}{\partial y}+c_1\right)\frac{\partial u}{\partial y}=0,\ (x,y)\in D_1 \tag{2}$$

in the domain D_1 has three valid characteristics: $x=const$ – one - time, $y=const$ – two - time characteristic. The equation

$$L_1L_2u \equiv \left(\frac{\partial^2}{\partial x \partial y}+c_2\right)\frac{\partial u}{\partial y}=0,\ (x,y)\in D_2 \tag{3}$$

in the domain D_2 has three valid characteristics: $x=const$ – two - time, and $y=const$ – one - time characteristic [1].

Let C^{n+m} means a class of functions having continuous all derivatives $\dfrac{\partial^{r+s}}{\partial x^r \partial y^s}\left(r=0,1,...,n;\ s=0,1,...,m\right)$ [2].

Problem 1. Find in the domain $D\backslash(y=0)$ function $u(x,y)$ from the class

$C(\bar{D}_i)\cap C^2(D_i)\cap[C^{2+2}(D_1)\cup C^{1+2}(D_2)]\,(i=1,2)$, being the solution of equation (1), satisfying the boundary conditions:

$$u\big|_{x=0}=\varphi_1(y),\ u\big|_{x=\ell}=\varphi_2(y),\ 0\le y\le h, \tag{4}$$

$$u\big|_{x=0}=\varphi_3(y),\ -h_1\le y\le 0, \tag{5}$$

$$u\big|_{y=-h_1}=\psi(x),\ 0\le x\le \ell, \tag{6}$$

and discontinuous bonding conditions

$$\begin{gathered} u(x,-0)=\alpha(x)u(x,+0)+\delta(x),0\le x\le\ell, \\ u_y(x,-0)=\beta(x)u_y(x,+0)+\chi(x),0\le x\le\ell, \\ u_{yy}(x,-0)=\gamma(x)u_{yy}(x,+0)+\rho(x),0\le x\le\ell, \end{gathered} \tag{7}$$

where $\varphi_1(y),\ \varphi_2(y),\ \varphi_3(y),\ \psi(x),\ \alpha(x),\ \beta(x),\ \gamma(x),\ \delta(x),\ \chi(x),\rho(x)-$ given functions, at that

$$\begin{gathered} \varphi_1(y)\in C^2[0,h],\ \varphi_2(y)\in C^1[0,h],\ \varphi_3(y)\in C^2[-h_1,0],\ \psi(x)\in C^1[0,\ell], \\ \alpha(x),\ \beta(x),\ \gamma(x),\ \delta(x),\ \chi(x),\ \rho(x)\in C[0,\ell],\ c_1\le 0. \end{gathered} \tag{8}$$

$$\forall\, x\in[0,\ell]:\alpha(x)\beta(x)\gamma(x)\ne 0. \tag{9}$$

$$\begin{gathered} \varphi_3(0)=\alpha(0)\varphi_1(0)+\delta(0),\ \varphi_3'(0)=\beta(0)\varphi_1'(0)+\chi(0), \\ \varphi_3''(0)=\gamma(0)\varphi_1''(0)+\rho(0),\ \varphi_3(-h_1)=\psi(0). \end{gathered} \tag{10}$$

Problem 1, in case, when $c_1=0,\alpha(x)\equiv 1,\beta(x)\equiv 1,\gamma(x)\equiv 1,\delta(x)\equiv 0,$ $\chi(x)\equiv 0,\rho(x)\equiv 0$ learned in work [3], and for equations with minor terms , it is considered in the work [4].

Note that boundary value problems for second- and third-order equations of the form $L_2u=0$, $L_2L_1u=0$ studied in the works [5, 6]. Boundary value problems for mixed-type equations with discontinuous bonding conditions are considered, for example, in [7-9].

Let's introduce a new function

$$\upsilon(x,y)=u_y(x,y)\,. \tag{11}$$

Then from equation (1) we have

$$\upsilon_{xx}-\upsilon_y+c_1\upsilon=0,\ (x,y)\in D_1, \tag{12}$$

$$\upsilon_{xy}+c_2\upsilon=0,\ (x,y)\in D_2, \tag{13}$$

Problem 2. Find in the domain $D\setminus(y=0)$ function $\upsilon(x,y)$ from the class $C(\bar{D}_i)\cap C^1(D_i)\cap[C^{2+1}(D_1)\cup C^{1+1}(D_2)]\ (i=1,2)$, satisfying in the domain of D_1 equation (12) and boundary conditions

$$\upsilon\big|_{x=0}=\varphi_1'(y),\ \upsilon\big|_{x=\ell}=\varphi_2'(y),\ 0\le y\le h, \tag{14}$$

satisfying in the domain of D_2 equation (13) and boundary conditions

$$\upsilon\big|_{x=0}=\varphi_3'(y),\ -h_1\le y\le 0, \tag{15}$$

and discontinuous bonding conditions

$$\begin{aligned}&\upsilon(x,-0)=\beta(x)\upsilon(x,+0)+\chi(x),0\le x\le\ell,\\&\upsilon_y(x,-0)=\gamma(x)\upsilon_y(x,+0)+\rho(x),0\le x\le\ell.\end{aligned} \tag{16}$$

Let $\upsilon(x,+0)=\nu_1(x),\ \upsilon_y(x,+0)=\mu_1(x),\upsilon(x,-0)=\nu_2(x),\upsilon_y(x,-0)=$ $=\mu_2(x),\ 0\le x\le\ell$. Then the condition (16) it will be written as

$$\begin{aligned}&\nu_2(x)=\beta(x)\nu_1(x)+\chi(x),0\le x\le\ell,\\&\mu_2(x)=\gamma(x)\mu_1(x)+\rho(x),0\le x\le\ell.\end{aligned} \tag{17}$$

2. Getting relationships between functions $\nu_2(x)$ and $\mu_2(x)$. Consider in the field D_2 Gursa 's problem, requiring finding a solution to the equation (13), satisfying the following boundary conditions

$$\upsilon(x,0)=\nu_2(x),0\le x\le\ell,\upsilon(0,y)=\psi_3'(y),-h_1\le y\le 0, \tag{18}$$

at that $\nu_2(0)=\psi_3'(0)$. The solution to this problem is represented as [3]:

$$\begin{aligned}&\upsilon(x,y)=\nu_2(x)+\varphi_3'(y)-R(x,y;0,0)\varphi_3'(0)-\\&-\int_0^x R_\xi(x,y;\xi,0)\nu_2(\xi)\,d\xi-\int_0^y R_\eta(x,y;0,\eta)\varphi_3'(\eta)\,d\eta,\end{aligned} \tag{19}$$

where

$$R(x,y;\xi,\eta)=1+c_2(x-\xi)(\eta-y)+c_2^2\frac{(x-\xi)^2(\eta-y)^2}{[2!]^2}+c_2^3\frac{(x-\xi)^3(\eta-y)^3}{[3!]^2}+$$

$$+\ldots+c_2^n\frac{(x-\xi)^n(\eta-y)^n}{[n!]^2}+\ldots=\sum_{n=0}^{+\infty}c_2^n\frac{(x-\xi)^n(\eta-y)^n}{[n!]^2}$$

- Riemann function of the equation (13), having the following properties:

$$R(0,0;0,0)=1, \qquad R_{\xi}(x,y;x,\eta)=-c_2(\eta-y), \qquad R_{\xi}(x,y;x,0)=c_2 y,$$

$$R_{\xi y}(x,y;x,0)=c_2,\ R_{\eta}(x,y;\xi,y)=-c_2(\xi-x), \qquad R_{\eta}(x,y;0,y)=c_2 x,$$

$$R_{\eta x}(x,y;0,y)=c_2, \qquad R_{xy}(x,0;0,0)=-c_2, \qquad R_{xy}(0,y;0,0)=-c_2,$$

$$R_{\xi\eta}(x,y;\xi,y)=-c_2 R(x,y;\xi,y),\ R_{\xi\eta\xi\eta}(x,y;\xi,y)=c_1^2 R(x,y;\xi,y),$$

Note that the Riemann function $R(x,y;\xi,\eta)$ it is representable through zero - order Bessel functions with imaginary and real arguments, respectively [10]:

$$R(x,y;\xi,\eta)=\begin{cases} I_0\left(2\sqrt{c_2}\sqrt{(x-\xi)(\eta-y)}\right),\ c_2>0, 0\le\xi\le x,\ y\le\eta\le 0, \\ J_0\left(2\sqrt{-c_2}\sqrt{(x-\xi)(\eta-y)}\right),\ c_2<0, 0\le\xi\le x,\ y\le\eta\le 0, \end{cases}$$

where $I_0\left(2\sqrt{c_2}\sqrt{(x-\xi)(\eta-y)}\right)=\sum\limits_{n=0}^{+\infty}\frac{1}{[n!]^2}\left(\frac{\sqrt{4c_2}\sqrt{(x-\xi)(\eta-y)}}{2}\right)^{2n}=\sum\limits_{n=0}^{+\infty}c_2^n\frac{(x-\xi)^n(\eta-y)^n}{[n!]^2},\ c_2>0$;

$$J_0\left(2\sqrt{-c_2}\sqrt{(x-\xi)(\eta-y)}\right)=\sum_{n=0}^{+\infty}\frac{1}{[n!]^2}\left(\frac{\sqrt{4(-c_2)}\sqrt{(x-\xi)(\eta-y)}}{2}\right)^{2n}=\sum_{n=0}^{+\infty}(-c_2)^n\frac{(x-\xi)^n(\eta-y)^n}{[n!]^2},\ c_2<0.$$

Differentiating the ratio (19) by y, we have:

$$\begin{aligned} \upsilon_y(x,y)=\varphi_3''(y)-R_y(x,y;0,0)\varphi_3'(0)-\int_0^y R_{\xi y}(x,y;\xi,0)\nu_2(\xi)\,d\xi- \\ -R_{\eta}(x,y;0,y)\varphi_3'(y)-\int_0^y R_{\eta y}(x,y;0,\eta)\varphi_3'(\eta)d\eta,\ (x,y)\in D_2. \end{aligned} \tag{20}$$

Aspiring $y\to-0$ and considering, that $R_{\eta}(x,y;0,y)=0$, $R_y(x,0;0,0)=-c_2 x$, $R_{\xi y}(x,y;\xi,0)=c_2$, from (21) we get the relation between the functions $\nu_2(x)$ and

$$\mu_2(x):\mu_2(x)=\varphi_3''(0)-c_2\int_0^x \nu_2(\xi)d\xi,\ 0\le x\le\ell. \tag{21}$$

Ratio (21) directly can be obtained from the equation (13).

3. Getting relationships between functions $\nu_1(x)$ and $\mu_1(x)$. Aspiring $y\to+0$ from the equations (12), we have the following relation:

$$\nu_1''(x)+c_1\nu_1(x)=\mu_1(x),\ 0\le x\le\ell. \tag{22}$$

For $\nu_1(x)$ we have the following boundary conditions:

$$\nu_1(0)=\varphi_1'(0),\ \ \nu_1(\ell)=\varphi_2'(0) \tag{23}$$

solution of problem (22), (23) by $c_1=-\lambda^2$ using the Green 's function, we represent as

$$\nu_1(x)=g_1(x)+\int_0^{\ell} K_1(x,\xi)\mu_1(\xi)dx\,, \tag{24}$$

where $K_1(x,\xi)=-\lambda^2\int_{\xi}^{x}G_1(x,t)\,dt,\ g_1(x)=\varphi_1''(0)+\dfrac{x}{\ell}\left[\varphi_2''(0)-\varphi_1''(0)\right]+\varphi_1''(0)\int_0^{\ell}G_1(x,t)\,dt$

$$G_1(x,t)=\begin{cases}\dfrac{sh\lambda x\, sh\lambda(\xi-\ell)}{\lambda sh\lambda\ell},\ 0\le x\le\xi,\\[2mm] \dfrac{sh\lambda\xi\, sh\lambda(x-\ell)}{\lambda sh\lambda\ell},\ \xi\le x\le\ell\end{cases}$$ – Green 's function [6].

From (21) and (24) taking into account (17), we come to the equation

$$\mu_1(x)=g(x)+\int_0^{\ell} K(x,\xi)\mu_1(\xi)dx\,, \tag{25}$$

where $K(x,\xi)=-\dfrac{c_2}{\gamma(x)}\int_{\xi}^{x}\beta(t)K_1(t,\xi)\,dt,$

$$g(x)=\frac{1}{\gamma(x)}\left[\varphi_3''(0)-\rho(x)-c_2\int_0^{x}\beta(t)g_1(t)\,dt--c_2\int_0^{x}\chi(t)\,dt\right].$$

When the condition is met $\ell\cdot\max\limits_{(x,\xi)\in[0,\ell]}|K(x,\xi)|\le 1$ (26)

the Fredholm integral equation of the second kind (25) admits a unique solution.

After the definition $\mu_1(x)$ from (24) we find $\nu_1(x)$. Then $\nu_2(x)$ determined from the first formula (17). Then from (21) find $\mu_2(x)$. Thus, the desired function $\upsilon(x,y)$ in the domain D_2 determined by the formula (16), and in the domain D_1 - as a solution to the first boundary value problem for the heat conduction equation in the form [6]:

$$\begin{aligned}\upsilon(x,y)&=\int_0^{x}G_{\xi}(x,y;0,\eta)\exp\left[-\lambda^2(y-\eta)\right]\varphi_1'(\eta)\,d\eta-\\&-\int_0^{x}G_{\xi}(x,y;\ell,\eta)\exp\left[-\lambda^2(y-\eta)\right]\varphi_2'(\eta)d\eta+\\&+\int_0^{\ell}G(x,y;\xi,0)\exp(-\lambda^2y)\nu_1(\xi)\,d\xi,\end{aligned} \tag{27}$$

where $G(x,y;\xi,\eta)$ – Green's function, presented in the form

$$G(x,y;\xi,\eta)=\sum_{n=-\infty}^{+\infty}\frac{1}{2\sqrt{\pi(y-\eta)}}\left[\exp\left(-\frac{(x-\xi-2n\ell)}{4(y-\eta)}\right)-\exp\left(-\frac{(x+\xi-2n\ell)}{2(y-\eta)}\right)\right].$$

4. Construction solution problem 1 in domains D_2 and D_1. After finding function $\upsilon(x,y)$ to determine the solution of problem 1 in the field D_2 we come to the next problem: find in the domain $D\setminus(y=0)$ solution of the equation

$$\begin{cases} u_y(x,y)=\upsilon(x,y),\ (x,y)\in D_1, \\ u_y(x,y)=\upsilon(x,y),\ (x,y)\in D_2, \end{cases} \tag{28}$$

satisfying boundary condition (6) and gluing condition:

$$\tau_2(x)=\alpha(x)\tau_1(x)+\delta(x), 0\le x\le \ell, \tag{29}$$

where $\tau_1(x)=u(x,+0), \tau_2(x)=u(x,-0)$. Condition (29) follows from the first gluing condition (7).

The solution of this problem in the field of D_2 has the form

$$u(x,y)=\psi(x)+\int_{-h_2}^{y}\upsilon(x,\eta)\,d\eta,\ (x,y)\in D_2\ . \tag{30}$$

Assuming $y=0$, form (30) define $\tau_2(x)$:

$$\tau_2(x)=\psi(x)+\int_{-h_2}^{0}\upsilon(x,\eta)\,d\eta,\ 0\le x\le \ell\ .$$

Тогда из (29) we find $\tau_1(x)=\dfrac{1}{\alpha(x)}\left[\tau_2(x)-\delta(x)\right], 0\le x\le \ell$. From the first equation (28) we find $u(x,y)=\tau_1(x)+\int_0^y \upsilon(x,\eta)d\eta,\ (x,y)\in D_1$.

Theorem 1. If the conditions are met (8), (9), (10) and (26), then the solution to problem 1 exists and it is the only one.

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ON A METRIC ON A HYPERSPACE

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In the paper we had constructed a function being a metric on the hyperspace of a metrizable space. Then it had established that this metric generates the Vietoris topology on the hyperspace. Further on the hyperspace of a uniform space we have allocated a (ε,δ)-uniform family of pseudo metrics, which generates a uniformity on the hyperspace.

Keywords: metric, hypespace, uniformity.

Бул макалада биз метрикалуу мейкиндиктин гипер мейкиндигинде метрика болгон функцияны курабыз. Бул метрика гипер мейкиндикте Vietoris топологиясын түзөрү аныкталган. Андан ары бир тектүү мейкиндиктин гипер мейкиндигинде ага бирдейликти жараткан псевдометриянын (ε,δ)-униформдуу үй-бүлөсү айырмаланат.

Урунтуу сөздөр: метрика, гипермейкиндик, бирдейлик.

В работе построена функция, являющаяся метрикой на гиперпространстве метризуемого пространства. Установлено, что эта метрика порождает топологию Вьеториса на гиперпространстве. Далее, на гиперпространстве равномерного пространства выделено (ε,δ)-равномерное семейство псевдометрик, порождающее равномерность на нем.

Ключевые слова: метрика, гиперпространство, равномерность.

As it is known, the Hausdorff metric evaluates how two sets differ in their emplacement in the metric space. Let (X,ρ) be a metric space. The Hausdorff distance (metric) ρ_H between two compact subsets F_1, F_2 of the space X is the number $\rho_H(F_1,F_2)=\max\{\inf\{\varepsilon:\varepsilon>0, F_2\subset O_\varepsilon F_1\}, \inf\{\varepsilon:\varepsilon>0, F_1\subset O_\varepsilon F_2\}\}$.

In this paper, we introduce another metric on the hyperspace, denoted by ρ_Z. For this purpose, we present the following constructions.

Let X be a compact Hausdorff space. Denote (see, [3])

$X_1=X_2=X_3=X$, $X_{12}=X_1\times X_2$, $X_{13}=X_1\times X_3$, $X_{23}=X_2\times X_3$,

$\Delta = \{(x,y) \in X^2 : x = y\}$, $X_{123} = X_1 \times X_2 \times X_3$.

Consider projections

$\pi_{12}^{123} : X_{123} \to X_{12}$, $\quad \pi_{13}^{123} : X_{123} \to X_{13}$, $\quad \pi_{23}^{123} : X_{123} \to X_{23}$,

$\pi_1^{12} : X_{12} \to X_1$, $\quad \pi_1^{13} : X_{13} \to X_1$, $\quad \pi_2^{23} : X_{23} \to X_2$,

$\pi_2^{12} : X_{12} \to X_2$, $\quad \pi_3^{13} : X_{13} \to X_3$, $\quad \pi_3^{23} : X_{23} \to X_3$.

Lemma 1. Let $\Phi_{12} \subset X_{12}$ and $\Phi_{23} \subset X_{23}$ be an arbitrary pair of subsets such that

$\pi_2^{12}(\Phi_{12}) = \pi_2^{23}(\Phi_{23})$.

Then there exists a set $\Phi_{123} \subset X_{123}$ such that

$\pi_3^{23} : X_{23} \to X_3$, $\qquad \pi_{23}^{123}(\Phi_{123}) = \Phi_{23}$ v

Proof. Putting $\Phi_{123} = \left(\pi_{12}^{123}\right)^{-1}(\Phi_{12}) \cap \left(\pi_{23}^{123}\right)^{-1}(\Phi_{23})$ we get the searching set.

Lemma 2. Let F_1, F_2 and F_3 be an arbitrary triple of subsets of X, and $\Phi_{12} \subset X_{12}$ and $\Phi_{23} \subset X_{23}$ sets such that

$\pi_1^{12}(\Phi_{12}) = F_1$, $\qquad \pi_2^{23}(\Phi_{23}) = F_2$,

$\pi_2^{12}(\Phi_{12}) = F_2$, $\qquad \pi_3^{23}(\Phi_{23}) = F_3$.

Then there exists a set $\Phi_{13} \subset X_{13}$ such that

$\pi_1^{13}(\Phi_{13}) = F_1$, $\qquad \pi_3^{13}(\Phi_{13}) = F_3$.

Proof. According to Lemma 1, we construct a set Φ_{123}, and then put $\Phi_{13} = \pi_{13}^{123}(\Phi_{123})$. This set has the required conditions.

For a compact Hausdorff space X let $\exp X$ be the hyperspace of X, i. e. a set of all closed nonempty subsets of X equipped with the Vietoris topology.

For a compact metric space X we define a function $\rho_Z : \exp X \times \exp X \to \mathbb{R}$ by the equality

$$\rho_Z(F_1, F_2) = \inf\left\{\sup\{\rho(x,y) : (x,y) \in \Phi\} : \Phi \in \exp X^2, \pi_1^{12}(\Phi) = F_1,\ \pi_2^{12}(\Phi) = F_2\right\}.$$

Lemma 3. For every couple of sets F_1, $F_2 \in \exp X$ there exists a set $\Phi_{12} \in \exp X^2$ such that $\pi_1^{12}(\Phi_{12}) = F_1$, $\pi_2^{12}(\Phi_{12}) = F_2$ and

$\rho_Z(F_1,F_2)=\sup\{\rho(x,y):(x,y)\in\Phi_{12}\}$.

Proof. Clearly, for the product $F_1\times F_2$ equalities $\pi_1^{12}(F_1\times F_2)=F_1$, $\pi_2^{12}(F_1\times F_2)=F_2$ hold. Let $b=\sup\{\rho(x,y):(x,y)\in F_1\times F_2\}$. Since F_1 and F_2 are compact and the metric ρ is a continuous function on $X\times X$, we have $b=\max\{\rho(x,y):(x,y)\in F_1\times F_2\}<\infty$. For every $\Phi\subset F_1\times F_2$ such that $\pi_1^{12}(\Phi)=F_1$, $\pi_2^{12}(\Phi)=F_2$ one has $0\le\sup\{\rho(x,y):(x,y)\in\Phi\}\le b$. So, for every net $\{\Phi^\alpha\}$ such that $\Phi^\alpha\subset F_1\times F_2$, $\pi_1^{12}(\Phi^\alpha)=F_1$, $\pi_2^{12}(\Phi^\alpha)=F_2$, $\alpha\in\mathfrak{A}$, the number net $\{\sup\{\rho(x,y):(x,y)\in\Phi^\alpha\}\}$ has a limit. Consider such a net $\{\Phi^\alpha\}$ which besides listed above properties satisfies the following condition: $\sup\{\rho(x,y):(x,y)\in\Phi^\alpha\ge\sup\{\rho(x,y):(x,y)\}\in\Phi^\beta\}$ for all α, β, $\alpha\prec\beta$. The number net $\{\sup\{\rho(x,y):\,(x,y)\in\Phi^\alpha\}\}$ has a limit. Now using the Zorn's lemma, we complete the proof.

Theorem 1. For any metric ρ on a compact metrizable space X the function ρ_Z is a metric on $\exp X$.

Proof. It is easy to see that $\rho_Z\ge 0$. For any $F\in\exp X$ we have

$$\rho_Z(F,F)=\inf\{\sup\{\rho(x,y):(x,y)\in\Phi\}:\Phi\in\exp X^2,\pi_1^{12}(\Phi)=F,\pi_2^{12}(\Phi)=F\}\le$$

$$\le\sup\{\rho(x,y):(x,y)\in\Delta\}=0,$$

i. e. $\rho_Z(F,F)=0$. Let now $\rho_Z(F_1,F_2)=0$ for some F_1, $F_2\in\exp X$. It should exist a nonempty closed subset $\Phi\subset X^2$ such that $\pi_1^{12}(\Phi)=F_1$, $\pi_2^{12}(\Phi)=F_2$ and $\Phi\subset\Delta$. The last inclusion implies $\pi_1^{12}(\Phi)=\pi_2^{12}(\Phi)$. Consequently, $F_1=F_2$.

Thus $\rho_Z(F_1,F_2)=0$ if and only if $F_1=F_2$.

Clearly, the function ρ_Z is symmetric.

It reminds to establish the triangle rule. Take any triple F_1, F_2, $F_3\in\exp X$. Then

$$\rho_Z(F_1,F_2)+\rho_Z(F_2,F_3)=\inf\{\sup\{\rho(x,y):(x,y)\in\Phi\}:\Phi\in\exp X^2,\pi_i^{12}(\Phi)=F_i,i=1,2\}$$

$$+\inf\left\{\sup\left\{\rho(y,z):(y,z)\in\Phi\right\}:\Phi\in\exp X^2,\pi_i^{23}(\Phi)=F_i,i=2,3\right\}$$
$$=(by\ \ force\ of\ \ Lemma\ 3)=\sup\left\{\rho(x,y):(x,y)\in\Phi_{12}\right\}+\sup\left\{\rho(y,z):(y,z)\in\Phi_{23}\right\}$$
$$=(by\ \ force\ of\ \ Lemma\ 1)=\sup\left\{\rho(x,y):(x,y,z)\in\Phi_{123}\right\}+\sup\left\{\rho(y,z):(x,y,z)\in\Phi_{123}\right\}$$
$$\geq(by\ the\ \ property\ of\ \ \sup)\geq\sup\left\{\rho(x,y)+\rho(y,z):(x,y,z)\in\Phi_{123}\right\}$$
$$\geq(by\ the\ triangle\ axiom)\geq\sup\left\{\rho(x,z):(x,y,z)\in\Phi_{123}\right\}$$
$$=(by\ \ force\ of\ \ Lemma\ 2)=\sup\left\{\rho(x,z):(x,z)\in\Phi_{13}\right\}$$
$$\geq(by\ the\ \ property\ of\ \ \inf)\geq\inf\left\{\sup\left\{\rho(x,z):(x,z)\in\Phi\right\}:\Phi\in\exp X^2,\pi_i^{13}(\Phi)=F_i,i=1,3\right\}$$
$$=(by\ the\ definition)=\rho_Z(F_1,F_3).$$

Theorem 1 is proved.

Theorem 2. For a compact metric space (X,ρ) the metric ρ_Z generates the Vietoris topology on $\exp X$.

Proof. Consider a sequence $\{F_n\}$ of closed sets converging in the Vietoris topology to a closed set F_0. Suppose $\lim\limits_{n\to\infty}\rho_Z(F_n,F_0)=a>0$. Without losing generality we can assume that $\rho_Z(F_n,F_0)\geq\frac{a}{2}$ for all $n\in\mathbb{N}$. We claim that $F_n\not\subset O_{\frac{a}{4}}F_0$ and $F_0\not\subset O_{\frac{a}{4}}F_n$. Otherwise we get an inequality: $\rho_Z(F_n,F_0)\leq\frac{a}{4}$. Hence, $F_n\not\subset O_{\frac{a}{4}}F_0$ and $F_0\not\subset O_{\frac{a}{4}}F_n$ for all n. Consequently, the sequence $\{F_n\}$ does not converges to F_0 in the Vietoris topology. We have obtained a contradiction.

Let us show the opposite. Let $\rho_Z(F_n,F_0)\to 0$. Take sets $\Phi_{n0}\in\exp X^2$ satisfying the conclusion of Lemma 3. Each set Φ_{n0} is assigned a number $\sup\left\{\rho(x,y):(x,y)\in\Phi_{n0}\right\}$. It is understandable that the number 0 corresponds to the only set $\Phi_{00}=\Delta(X)\cap(F_0\times F_0)$. According to the data, the sequence $\{\sup\left\{\rho(x,y):(x,y)\in\Phi_{n0}\right\}\}_{n=1}^{\infty}$ has to converge to 0. Then for any $\varepsilon>0$ there exists n_0 such that $F_n\subset O_\varepsilon F_0$ for all $n\geq n_0$. So, the sequence $\{F_n\}$ converges to F_0 in the Vietoris topology.

Theorem 2 is proved.

Lemma 4. For any pseudometric ρ on X the function ρ_Z is a pseudometric on $\exp X$.

Proof. Since the proof of the Lemma consists of repeating the reasonings carried out in the proof of Theorem 1, it suffices to note that in this case the function ρ_Z is not a metric on $\exp X$. Indeed, let x, $y \in X$, $x \neq y$ and $\rho(x,y)=0$. Then $\rho_Z(\{x\},\{y\})=\rho(x,y)=0$ although $\{x\}\neq\{y\}$.

Consider a Tychonoff space X and define a set (see, [2], [5])

$\exp X = \{K \subset X:\ K \textit{ is compact and } K \neq \varnothing\}$.

Consider any family $\mathfrak{P}$ of pseudometrics and define

$U_d(r)=\{(x,y):\, d(x,y)<r\}$

for $d \in \mathfrak{P}$ and $r>0$. The resulting family $\{U_d(r):\, d\in\mathfrak{P}, r>0\}$ of entourages is a subbase for a uniformity in that the family of finite intersections is a base for a uniformity, denoted $\mathcal{U}_{\mathfrak{P}}$.

Given a sequence $\{V_n : n\in\mathbb{N}\}$ of entourages on a set X such that $V_0 = X^2$ and $V_{n+1}^3 \subset V_n$ for all n one can find a pseudometric d on X such that $U_d(2^{-n})\subset V_n \subset \{(x,\ y): d(y,x)\le 2^{-n}\}$ for all n. Thus every uniform structure can be defined by a family of pseudometrics. The family of all pseudometrics d that satisfy $(\forall r>0)(U_d(r)\in\mathcal{U})$ is denoted $\mathfrak{P}_{\mathcal{U}}$; it is the largest family of pseudometrics that generate $\mathcal{U}$ [1], [4]. The family $\mathfrak{P}_{\mathcal{U}}$ satisfies the following two properties.

$(P1)$ If d_1, $d_2 \in \mathfrak{P}_{\mathcal{U}}$ then $\max\{\rho_1,\rho_2\}\in\mathfrak{P}_{\mathcal{U}}$;

$(P2)$ If d is a pseudometric and for every $\varepsilon>0$ there are $\rho\in\mathfrak{P}_{\mathcal{U}}$ and $\delta>0$ such that always $\rho(x,y)<\delta$ implies $d(x,y)<\varepsilon$ then $d\in\mathfrak{P}_{\mathcal{U}}$.

A family $\mathfrak{P}$ of pseudometrics with these properties is called a pseudometric uniformity; it satisfies the equality $\mathfrak{P}_{\mathcal{U}_{\mathfrak{P}}}=\mathfrak{P}$ [4].

Theorem 3. For a Tychonoff space X and a family of pseudometrics $\mathfrak{P}=\{\rho\}$ generating a uniformity on X the family

$$\mathfrak{P}_{\exp}=\{d : d\, is\ a\ \ pseudometric\ on \exp X\ and$$

$$\forall\varepsilon>0,\ \exists\rho\in\mathfrak{P},\ \exists\delta>0\ \ that\ \ \rho_Z\left(F_1,F_2\right)<\delta\Rightarrow d\left(F_1,F_2\right)<\varepsilon\}$$

generates a uniformity on $\exp X$.

Proof. We will show that the family $\mathfrak{P}_{\exp}$ satisfies the above two conditions (P1) and (P2).

Let $\rho_1,\ \rho_2\ \in\mathfrak{P}$ be arbitrary pseudometrics and $F_1,\ F_2\in\exp X$ arbitrary sets. Take sets Φ_{12}^1, Φ_{12}^2 and $\Phi_{12}^{\vee}$ satisfying Lemma 3 for ρ_1, ρ_2 and $\max\{\rho_1,\rho_2\}$, respectively. We have

$$\max\{\rho_{1Z},\rho_{2Z}\}(F_1,F_2)=\max\{\rho_{1Z}\left(F_1,F_2\right),\rho_{2Z}\left(F_1,F_2\right)\}$$

$$=\max\left\{\inf_{\Phi^1}\sup_{(x,y)\in\Phi^1}\rho_1(x,y),\inf_{\Phi^2}\sup_{(x,y)\in\Phi_{12}^2}\rho_2(x,y)\right\}$$

$$=\max\left\{\sup_{(x,y)\in\Phi_{12}^1}\rho_1(x,y),\ \sup_{(x,y)\in\Phi_{12}^2}\rho_2(x,y)\right\}$$

$$\le\max\left\{\sup_{(x,y)\in\Phi_{12}^{\vee}}\rho_1(x,y),\ \sup_{(x,y)\in\Phi_{12}^{\vee}}\rho_2(x,y)\right\}$$

$$=\sup_{(x,y)\in\Phi_{12}^{\vee}}\{\max\{\rho_1,\rho_2\}(x,y)\}$$

$$=(\max\{\rho_1,\rho_2\})_Z\left(F_1,F_2\right),$$

i. e. $\max\{\rho_{1Z},\rho_{2Z}\}(F_1,F_2)\le(\max\{\rho_1,\rho_2\})_Z\left(F_1,F_2\right)$.

Let us show the reverse inequality. One can check that the following are valid.

$$(\max\{\rho_1,\rho_2\})_Z\left(F_1,F_2\right)=\inf_{\Phi}\sup_{(x,y)\in\Phi}\max\{\rho_1,\rho_2\}(x,y)$$

$$=\inf_{\Phi}\sup_{(x,y)\in\Phi}\max\{\rho_1(x,y),\rho_2(x,y)\}$$

$$=\inf_{\Phi}\max\left\{\sup_{(x,y)\in\Phi}\rho_1(x,y),\sup_{(x,y)\in\Phi}\rho_2(x,y)\right\}$$

$$\le \inf_{\Phi} \max\left\{ \sup_{(x,y)\in\Phi_{12}^{1}} \rho_1(x,y), \sup_{(x,y)\in\Phi} \rho_2(x,y) \right\}$$

$$\le \max\left\{ \sup_{(x,y)\in\Phi_{12}^{1}} \rho_1(x,y), \sup_{(x,y)\in\Phi_{12}^{2}} \rho_2(x,y) \right\}$$

$$= \max\{\rho_{1Z}(F_1,F_2), \rho_{2Z}(F_1,F_2)\} = \max\{\rho_{1Z},\rho_{2Z}\}(F_1,F_2).$$

Due to the arbitrariness of the sets F_1 and F_2 we get $\max\{\rho_{1Z},\rho_{2Z}\} = (\max\{\rho_1,\rho_2\})_Z$. On the other hand $\max\{\rho_1,\rho_2\} \in \mathfrak{P}$. So, by construction we obtain that $\max\{\rho_{1Z},\rho_{2Z}\} \in \mathfrak{P}_{\exp}$.

For any two pseudometrics d_1, $d_2 \in \mathfrak{P}_{\exp}$ suppose

$\forall \varepsilon > 0,\ \exists \rho_i \in \mathfrak{P},\ \exists \delta_i > 0$ that $\rho_{iz}(F_1,F_2) < \delta_i \Rightarrow d_i(F_1,F_2) < \varepsilon,\ i = 1,2;\ F_1,F_2 \in \exp X$.

Then

$\max\{\rho_{1Z},\rho_{2Z}\}(F_1,F_2) < \min\{\delta_1,\delta_2\} \Rightarrow \max\{d_1,d_2\}(F_1,F_2) < \varepsilon,\ i = 1,2;\ F_1,F_2 \in \exp X$.

Consequently, $\max\{d_1,d_2\} \in \mathfrak{P}_{\exp}$.

there exist pseudometrics ρ_1, $\rho_2 \in \mathfrak{P}$ such that $d_1 \le \rho_{1Z}$ and $d_2 \le \rho_{2Z}$. Then $\max\{d_1,d_2\} \le \max\{\rho_{1Z},\rho_{2Z}\}$. Hence, $\max\{d_1,d_2\} \in \mathfrak{P}_{\exp}$ Condition (P1) is verified.

Let now d be a pseudometric on $\exp X$ such that for every $\varepsilon > 0$ there are $\tilde{d} \in \mathfrak{P}$ and $\delta > 0$ such that $\tilde{d}_Z(F_1,F_2) < \delta \Rightarrow d(F_1,F_2) < \varepsilon, \quad F_1,F_2 \in \exp X$.

Then by construction there exists $d \in \mathfrak{P}_{\exp}$. Condition (P2) is established, and the proof of Theorem 3 is completed.

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ON SUFFICIENT CONDITIONS FOR THE INITIAL PROBLEM FOR NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS IN PARTIAL DERIVATIVES

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The solvability of the initial problem and the structure of solutions for nonlinear integro-differential partial differential equations are studied.

Keywords: solution transformation method, nonlinear partial differential integro-differential equations, Cauchy problem, contraction mapping principle, Lipschitz conditions.

Сызыктуу эмес жекече туундулуу интегро-дифференциалдык теңдемелерге коюлган Коши маселесинин чыгарымдуулугу жана чыгарылыштарынын түзүлүшү изилденди.

Урунттуу сөздөр: чыгарылышты өзгөртүп түзүү усулу, сызыктуу эмес жекече туундулуу интегро-дифференциалдык теңдемелер, Коши маселеси, кысып чагылтуу принциби, Липшиц шарттары.

Исследованы разрешимость начальной задачи и структура решений для нелинейных интегро-дифференциальных уравнений в частных производных.

Ключевые слова: метод преобразования решений, нелинейные интегро-дифференциальные уравнения в частных производных, задача Коши, принцип сжатых отображений, условия Липшица.

1. Introduction

Integro-differential equations in partial derivatives, which allow mathematical representation of processes with aftereffect occurring in space and time, play an important role in mathematics and its applications. The main theories of partial differential equations, problems of mathematical physics leading to partial differential equations, the fundamental general theory of difference schemes, in the field of applications of differential equations to continuum mechanics and with the development of numerical methods for solving these equations by the authors in [1-9] are described. Until now, the problem of finding out the solvability of the Cauchy problem for integro-differential equations in partial derivatives remains a little-studied area.

In works [10-12], various questions of the solvability of the Cauchy problem for integro-differential, partial differential equations, and systems of equations were studied using the method of transforming solutions. The essence of this approach is to transform the original Cauchy problem into an integral equation equivalent to it, to which one can apply the topological method - the principle of compressed mappings.

In [13], researches to problems of existence and uniqueness of solutions to initial problems for linear Volterra integro-differential equations of convolution type in Banach spaces.

In [14], the question of the solvability of the Cauchy problem for systems of nonlinear integro-differential partial differential equations of the first order with a parameter was studied and an integral representation of the obtained solutions was found.

In recent years, interest has increased in the study of partial differential equations, fractional derivatives generalizing classical partial differential equations such as heat equation, wave equation, etc. In the works [15-16], the author obtained the results of studying the Cauchy problem for a linear differential equation with a fractional partial derivative of Caputo.

In [17-20], the solvability and structure of solutions of the Cauchy problem for partial differential equations were found, where an analytical method for constructing solutions of the classical Cauchy problem was proposed for them. The essence of the proposed method is to transform the solutions of the original Cauchy problem into an equivalent Volterra integral equation, to which the principle of contracted mappings is applicable.

In this section, we study the solvability of the Cauchy problem and the structure of solutions for non-linear partial differential equations

$$u_{txx}+\alpha u_{xx}+2\beta u_{tx}+2\alpha\beta u_x+\left(\beta^2+1\right)u_t+\alpha\left(\beta^2+1\right)u=$$

$$=f(t,x,u(t,x))+\int_0^t K(t,s,u(s))ds\,, \tag{1}$$

with initial condition $u(0,x)=\varphi(x)\,,$ (2)

where $\alpha,\ \beta$ are some positive constants, $f(t,x,u(t,x))$, $K(t,s,u)$ are known continuous functions; $\varphi(x)$ is a known continuous function uniformly bounded together with its derivatives included in (1).

The solution of the Cauchy problem (1)-(2) will be sought in the form

$$u(t,x)=c(t,x)+\int_0^t e^{-\alpha(t-s)}\int_{-\infty}^x e^{-\beta(x-\rho)}\sin(x-\rho)Q(s,\rho)d\rho ds\,, \tag{3}$$

where $c(t,x)$ is a known continuous function, while

$c(o,x)=\varphi(x)\,,$

$Q(t,x)$ is a new unknown function that needs to be determined [10, 215].

Assume that the function $c(t,x)$ is uniformly bounded along with its derivatives in (1).

To determine the function $Q(t,x)$, it is necessary to substitute (3) into (1). For this purpose, taking into account (3), we successively calculate the following relations.

We get

$$u_t=c_t+\int_{-\infty}^x e^{-\beta(x-\rho)}sin(x-\rho)Q(t,\rho)d\rho-\alpha(u-c). \tag{4}$$

Differentiating both parts of (4) with respect to x, we have

$$u_{tx}=c_{tx}-\beta\int_{-\infty}^x e^{-\beta(x-\rho)}sin(x-\rho)Q(t,\rho)d\rho+$$

$$+\int_{-\infty}^x e^{-\beta(x-\rho)}\cos(x-\rho)Q(t,\rho)d\rho-(u_x-c_x)\,.$$

From here, we get

$$u_{tx}+\alpha u_x=c_{tx}+\alpha c_x-\beta\left(u_t+\alpha u-c_t-\alpha c\right)+$$

$$+\int_{-\infty}^{x} e^{-\beta(x-\rho)}\cos(x-\rho)Q(t,\rho)d\rho\,; \qquad (5)$$

$$\alpha u_{xx}+u_{txx}+\beta u_{tx}+\alpha\beta u_x=c_{txx}+\alpha c_{xx}+\beta c_{tx}+\alpha\beta c_x+Q(t,x)-$$

$$-\beta\int_{-\infty}^{x} e^{-\beta(x-\rho)}\cos(x-\rho)Q(t,\rho)d\rho-\int_{-\infty}^{x} e^{-\beta(x-\rho)}\sin(x-\rho)Q(t,\rho)d\rho\,, \qquad (6)$$

where relation (5) is taken into account in the calculation. Adding term by term relations (4)-(6) we have

$$u_{txx}+\alpha u_{xx}+\beta u_{tx}+\alpha\beta u_x+\beta u_{tx}+\alpha\beta u_x+\beta^2 u_t+\beta^2\alpha u=$$

$$=c_{txx}+\alpha c_{xx}+2\beta c_{tx}+2\alpha\beta c_x+Q(t,x)+\beta^2 c_t+\beta^2\alpha c-\left(u_t+\alpha u-c_t-\alpha c\right)$$

or

$$u_{txx}+\alpha u_{xx}+2\beta u_{tx}+2\alpha\beta u_x+(\beta^2+1)u_t+\alpha\left(\beta^2+1\right)u=$$

$$=c_{tx}+\alpha c_{xx}+2\beta c_{tx}+2\alpha\beta c_x+Q(t,x)+(\beta^2+1)c_t+\alpha\left(\beta^2+1\right)c\,. \qquad (7)$$

Replacing the right side of (1) with the equation obtained from (7), we have a nonlinear integral equation for $Q(t,x)$

$$Q(t,x)=f\left(t,x,c+\int_0^t e^{-\alpha(t-s)}\int_{-\infty}^{x} e^{-\beta(x-\rho)}\sin(x-\rho)Q(s,\rho)d\rho ds\right)+$$

$$+\int_0^t K\left(\tau,s,c+\int_0^\tau e^{-\alpha(t-s)}\int_{-\infty}^{x} e^{-\beta(x-\rho)}\sin(x-\rho)Q(s,\rho)d\rho ds\right)d\tau+$$

$$+H(t,c)\,, \qquad (8)$$

where

$$H(t,c)=c_{tx}+\alpha c_{xx}+2\beta c_{tx}+2\alpha\beta c_x+(\beta^2+1)c_t+\alpha(\beta^2+1)c\,.$$

Nonlinear integral equation (8) will be solved by the method of contraction mapping principle.

Suppose:

1) For all $\{0\le t\le T,\ -\infty<c<\infty\}$, the function $H(t,c)$ is continuous and bounded

$\|H(t,c)\|\le M_0=const$;

2) in the domain $R=\{0\le t<\infty,\ -\infty<x<u<\infty\}$ the function $f(t,x,u)$ is continuous

and bounded

$$\| f(t,x,u)\| \le M_1 = const\,. \tag{9}$$

In addition, in this region, the function $f(t,x,u)$ satisfies the Lipschitz condition on the third argument u:

$$\| f(t,x,u_2) - f(t,x,u_1)\| \le L\|u_2 - u_1\|, \tag{10}$$

$$\|K(t,s,u_2) - K(t,s,u_1)\| \le L_1\|u_2 - u_1\|,$$

where L, L_1 are some positive constants.

We consider the right side of (8) as an operator $A[Q]$ acting on the function $Q(t,x)$ in the set $\Omega = \{Q(t,x):\ Q(t,x) \in C(R), \|Q(t,x)\| \le M\}$.

We have

$$\|A[Q]\| \le \| f(t,x,u)\| + M_1 + M_0 + TM_2 \le M$$

where $M = const$.

So, the operator $A\Omega$ maps the set Ω into itself: $A:\Omega \to \Omega$. Let us prove that A is a contraction operator.

Taking into account (8)-(10), we estimate the difference

$$\|A[Q_1(t,x)] - A[Q_2(t,x)]\| \le \left\| f\left(t,x,c + \int_0^t e^{-\alpha(t-s)} \int_{-\infty}^{x} e^{-\beta(x-\rho)} \sin(x-\rho) Q_1(t,s)\, d\rho ds \right) - \right.$$

$$\left. - f\left(t,x,c + \int_0^t e^{-\alpha(t-s)} \int_{-\infty}^{x} e^{-\beta(x-\rho)} \sin(x-\rho) Q_2(t,s)\, d\rho ds \right) \right\| +$$

$$+ \left\| \int_0^t \left[K\left(t,s,c + \int_0^t e^{-\alpha(t-s)} \int_{-\infty}^{x} e^{-\beta(x-\rho)} \sin(x-\rho) Q_1(s,\rho)\, d\rho ds \right) dt - \right.\right.$$

$$\left.\left. - K\left(\tau,s,c + \int_0^\tau e^{-\alpha(t-s)} \int_{-\infty}^{x} e^{-\beta(x-\rho)} \sin(x-\rho) Q_2(s,\rho)\, d\rho ds \right) \right] d\tau \right\| \le$$

$$\le L\left\{ \int_0^t e^{-\alpha(t-s)} \int_{-\infty}^{x} e^{-\beta(x-\rho)} \sin(x-\rho) \|Q_1(t,s) - Q_2(t,s)\|\, d\rho ds \right\} \le$$

$$\le \left(L\frac{1}{\alpha\beta} + TL_1\frac{1}{\alpha\beta} \right) \|Q_1(t,x) - Q_2(t,x)\| \le \frac{1}{2}\|Q_1(t,x) - Q_2(t,x)\|,$$

where α, β - are chosen such that

$$\frac{L+TL_1}{\alpha\beta}\le\frac{1}{2}$$

is fulfilled. It follows that the operator A is a contraction operator in the set Ω. Then, according to the contraction mapping principle, it follows that the non-linear integral equation (8) for all $\{0\le t\le T,\ -\infty<x<\infty\}$ has a unique continuous solution $Q(t,x)$.

We now investigate the differential properties of solutions to the Cauchy problem (1)-(2). From (3) for all $\{0\le t\le T,\ -\infty<x<\infty\}$ we have inequality

$$\|u(t,x)\|\le\|c(t,x)\|+\int_0^t e^{-\alpha(t-s)}\int_{-\infty}^{x} e^{-\beta(x-\rho)}|\sin(x-\rho)|\|Q(s,\rho)\|d\rho ds\le$$

$$\le c_0+\frac{M}{\alpha\beta}=M_{00}=const\,.$$

From (4) we have the estimate

$$\|u_t\|\le\|c_t\|+\int_{-\infty}^{x} e^{-\beta(x-\rho)}\sin(x-\rho)Q(t,\rho)d\rho+\|\alpha(u-c)\|\le\overline{M}_0\,.$$

Thus, the following theorem has been proved.

Theorem. Let 1) for all $\{0\le t\le T,\ -\infty<x<\infty\}$ the functions $c(t,x),\ c(0,x)=\varphi(x)$ are uniformly bounded together with their derivatives included in (1);

2) in the domain $R=\{0\le t<T,-\infty<x,u<\infty\}$ the function $f(t,x,u)$ is continuous and bounded.

$$\|f(t,x,u)\|\le M_1=const,$$

and for all $\{t\ge 0,\ -\infty<x<\infty\}$ the functions $H(t,c)$, $K(t,s,u)$ are continuous and bounded

$$\|H(t,c)\|\le M_0=const;\quad \|K(t,s,u(s))\|\le M_2=const.$$

In addition, in this region *R*, the functions $f(t,x,u)$, $K(t,s,u)$ satisfy the Lipschitz condition

$$\|f(t,x,u_2)-f(t,x,u_1)\|\le L\|u_2-u_1\|;$$

$$\|K(t,s,u_2)-K(t,s,u_1)\|\le L_1\|u_2-u_1\|$$

and

$$\frac{L+TL_1}{\alpha\beta}\leq\frac{1}{2}.$$

Then, for all $\{0\leq t\leq T, -\infty<x<\infty\}$, the nonlinear integro-differential equation (1) with initial data (2) has a unique limited solution. In addition, all derivatives included in equation (1) are uniformly bounded.

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ON THE DEVELOPMENT OF THE PARTIAL CUTTING METHOD FOR A LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION OF THE FIRST ORDER

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Sufficient conditions of boundedness on the semiaxis of all solutions of a linear Volterra integro-differential equation of the first order with two kernels are established by the development of the partial cutting method by cutting separately by the first argument of the first kernel and by the second argument of the second kernel. An illustrative example is constructed.

Keywords: integro-differential equation, boundedness, partial cutting of the kernel by the first argument, partial cutting of the kernel by the second argument.

Эки ядролуу сызыктуу Вольтерра тибиндеги интегро-дифференциалдык теңдеменин бардык чыгарылыштарынын жарым окто чектелгендигинин жетиштүү шарттары биринчи ядрону биринчи аргументи боюнча, экинчи ядрону экинчи аргументи боюнча жекече кесүү методун өнүктүрүү аркылуу табылат. Иллюстративдик мисал тургузулат.

Урунттуу сөздөр: интегро-дифференциалдык теңдеме, чектелгендик, ядрону биринчи аргументи бюнча жекече кесүү, ядрону экинчи аргументи бюнча жекече кесүү.

Устанавливаются достаточные условия ограниченности на полуоси всех решений линейного вольтеррова интегро-дифференциального уравнения первого порядка с двумя ядрами, развитием метода частичного срезывания посредством срезывания отдельно по первому аргументу первого ядра и по второму аргументу второго ядра. Строится иллюстративный пример.

Ключевые слова: интегро-дифференциальное уравнение, ограниченность, частичное срезывание ядра по первому аргументу, частичное срезывание ядра по второму аргументу.

All the functions involved are continuous and the relations take place when $t \geq t_0, t \geq \tau \geq t_0$; $J = [t_0, \infty)$; IDE - integro-differential equation.

Problem. Establish sufficient boundedness conditions on the half-interval J of all solutions of the following first-order IDE of the form :

$$x'(t) + a(t)x(t) + \int_{t_0}^{t} [K_1(t,\tau) + K_2(t,\tau)]x(\tau)d\tau = f(t), \quad t \ge t_0 \tag{1}$$

by developing the method of partial cutting [1-5] by cutting separately on the first argument t of the kernel $K_1(t,\tau)$ and on the second argument τ of the kernel $K_2(t,\tau)$.

It should be noted that earlier in many works, in particular, in [1-5], the method of partial cutting of the kernels of the considered IDEs by the second argument was developed. This problem is being solved for the first time.

Let $\psi(t)$ be some function [6, p.41]. Let 's introduce the notation: .

$R_1(t,\tau) \equiv (\psi(t))^{-1} K_1(t,\tau)$ $R_2(t,\tau) \equiv K_2(t,\tau)(\psi(\tau))^{-1}$.

In the future, we will need the following 2 lemmas.

Lemma 1. The following relation holds:

$$\int_{t_0}^{t} x(s)[\int_{t_0}^{s} K_1(s,\tau)x(\tau)d\tau]ds = [\int_{t_0}^{t} R_1(t,\tau)x(\tau)d\tau]X(t,t_0) - \frac{1}{2}T_1(t)(X(t,t_0))^2 +$$

$$+\frac{1}{2}\int_{t_0}^{t} T_1'(s)(X(s,t_0))^2 ds - \int_{t_0}^{t}[\int_{t_0}^{s} R_{1s}'(s,\tau)x(\tau)d\tau]X(s,t_0)ds, \tag{2}$$

where

$$X(t,t_0) \equiv \int_{t_0}^{t} \psi(\eta)x(\eta)d\eta, \ T_1(t) \equiv R_1(t,t)(\psi(t))^{-1} \equiv K_1(t,t)(\psi(t))^{-2}.$$

Proof. We introduce a cutting function $\psi(t)$ and by integrating in parts we have

$$\int_{t_0}^{t} x(s)[\int_{t_0}^{s} K_1(t,\tau)x(\tau)d\tau]ds = \int_{t_0}^{t}[\int_{t_0}^{s}(\psi(s))^{-1}K_1(s,\tau)x(\tau)d\tau]\psi(s)x(s)ds =$$

$$= \int_{t_0}^{t}[\int_{t_0}^{s} R_1(s,\tau)x(\tau)d\tau](\int_{t_0}^{s}\psi(\eta)x(\eta)d\eta)'ds = [\int_{t_0}^{t} R_1(t,\tau)x(\tau)d\tau]X(t,t_0) -$$

$$-\int_{t_0}^{t}[R_1(s,s)x(s)+\int_{t_0}^{s}R'_{1s}(s,\tau)x(\tau)d\tau]X(s,t_0)ds=[\int_{t_0}^{t}R_1(t,\tau)x(\tau)d\tau]X(t,t_0)-$$

$$-\int_{t_0}^{t}R_1(s,s)(\psi(s))^{-1}(X(s,t_0))'X(s,t_0)ds-\int_{t_0}^{t}[\int_{t_0}^{s}R'_{1s}(s,\tau)x(\tau)d\tau]X(s,t_0)ds=$$

$$=[\int_{t_0}^{t}R_1(t,\tau)x(\tau)d\tau]X(t,t_0)-\frac{1}{2}\int_{t_0}^{t}T_1(s)d_s(X(s,t_0))^2-\int_{t_0}^{t}[\int_{t_0}^{s}R'_{1s}(s,\tau)x(\tau)d\tau]X(s,t_0)ds.$$

Integration by parts in the second term, hence we arrive at the required relation (2).

The following lemma follows from the lemma of the article [1].

Lemma 2. The following relation is true:

$$\int_{t_0}^{t}x(s)[\int_{t_0}^{s}[K_2(s,\tau)x(\tau)d\tau]ds=\frac{1}{2}T_2(t)(X(t,t_0))^2-\frac{1}{2}\int_{t_0}^{t}T'_2(s)(X(s,t_0))^2ds-$$

$$-\int_{t_0}^{t}[\int_{t_0}^{s}R'_{2\tau}(s,\tau)X(\tau,t_0)d\tau]ds,$$

where $T_2(t)\equiv R_2(t,t)(\psi(t))^{-1}\equiv K_2(t,t)(\psi(t))^{-2}$.

For any solution $x(t)$ we multiply IDE (1) by $x(t)$, then integrate it within the range from t_0 to t, including in parts, while introducing the cutting function $\psi(t)$ and functions $R_1(t,\tau), R_2(t,\tau),$ we apply Lemmas 1, 2. Then we get the following identity:

$$(x(t))^2+2\int_{t_0}^{t}a(s)(x(s))^2ds+2[\int_{t_0}^{t}R_1(t,\tau)x(\tau)d\tau]X(t,t_0)-T_1(t)(X(t,t_0))^2+$$

$$+\int_{t_0}^{t}T'_1(s)(X(s,t_0))^2ds-2\int_{t_0}^{t}[\int_{t_0}^{s}R'_{1s}(s,\tau)x(\tau)d\tau]X(s,t_0)ds+T_2(t)(X(t,t_0))^2-$$

$$-\int_{t_0}^{t}T'_2(s)(X(s,t_0))^2ds-2\int_{t_0}^{t}[\int_{t_0}^{s}R'_{2\tau}(s,\tau)X(\tau,t_0)d\tau]x(s)ds\equiv$$

$$\equiv(x(t_0))^2+2\int_{t_0}^{t}f(s)x(s)ds,\ t\geq t_0\,. \tag{3}$$

Let $a(t)\geq 0,$ there be a number $\varepsilon>0$. Then, applying the well-known inequality $2u_1u_2\leq\frac{1}{\varepsilon}u_1^2+\varepsilon u_2^2$ $(u_1,u_2\in R)$ and the Cauchy-Bunyakovsky inequality, we transform the third term from the left side of identity (3) as follows:

$$2[\int_{t_0}^{t} R_1(t,\tau)x(\tau)d\tau]X(t,t_0)] \le \frac{1}{\varepsilon}(\int_{t_0}^{t} R_1(t,\tau)x(\tau)d\tau)^2 + \varepsilon(X(t,t_0))^2 =$$

$$= \frac{1}{\varepsilon}(\int_{t_0}^{t} R_1(t,\tau)(a(\tau))^{-\frac{1}{2}}(a(\tau))^{\frac{1}{2}}x(\tau)d\tau)^2 + \varepsilon(X(t,t_0))^2 \le$$

$$\le \frac{1}{\varepsilon}(\int_{t_0}^{t} R_1^2(t,\tau)(a(\tau))^{-1}d\tau)(\int_{t_0}^{t} a(\tau)(x(\tau))^2 d\tau) + \varepsilon(X(t,t_0))^2. \tag{4}$$

Also, we transform the integral from the right side of the identity (3):

$$\int_{t_0}^{t} f(s)x(s)ds = 2\int_{t_0}^{t} f(s)(a(s))^{-\frac{1}{2}}(a(s))^{\frac{1}{2}}x(s)ds \le$$

$$\le (\int_{t_0}^{t} (f(s))^2(a(s))^{-1}d\tau) + \int_{t_0}^{t} a(s)(x(s))^2 ds. \tag{5}$$

Taking into account transformations (4), (5) from identity (3), we will have the following inequality:

$$(x(t))^2 + [1 - \frac{1}{\varepsilon}\int_{t_0}^{t} R_1^2(t,\tau)(a(\tau))^{-1}d\tau](\int_{t_0}^{t} a(s)(x(s))^2 ds) + T(t)(X(t,t_0))^2 -$$

$$-\int_{t_0}^{t} T'(s)(X(s,t_0))^2 ds - 2\int_{t_0}^{t}[\int_{t_0}^{s} R'_{1s}(s,\tau)x(\tau)d\tau]X(s,t_0)ds -$$

$$-2\int_{t_0}^{t}[\int_{t_0}^{s} R'_{2\tau}(s,\tau)X(\tau,t_0)d\tau]x(s)ds \le (x(t_0))^2 + \int_{t_0}^{t}(f(s))^2(a(s))^{-1}ds, \tag{6}$$

где $T(t) \equiv T_2(t) - T_1(t) - \varepsilon.$

Theorem. 1) $a(t) \ge 0;$ 2) there exists $\varepsilon > 0$ such that

$$\int_{t_0}^{t} R_1^2(t,\tau)(a(\tau))^{-1}d\tau \le \varepsilon, T(t) > 0,$$

there exists a function $T^*(t) \in L^1(J, R_+)$ such that $T'(t) \le T^*(t)T(t);$

3) $\int_{t_0}^{t}[| R'_{1t}(t,\tau)|(T(t))^{-\frac{1}{2}} + | R'_{2\tau}(t,\tau)|(T(\tau))^{-\frac{1}{2}}]d\tau + (f(t))^2(a(t))^{-1} \in L^1(J, R_+).$

Then any solution of IDE (1) is bounded on the half-interval *J*.

Proof. Let 's introduce the notation:

$$u(t) \equiv (x(t))^2 + \frac{1}{\varepsilon}[\varepsilon - \int_{t_0}^{t} R_1^2(t,\tau)(a(\tau))^{-1} d\tau](\int_{t_0}^{t} a(s)(x(s))^2 ds) + T(t)(X(t,t_0))^2.$$

Then by subject to conditions 1, 2) of the theorem we have $u(t) \geq 0$ and from inequality (6) we obtain the following integral inequality:

$$u(t) \leq c_* + \int_{t_0}^{t} \{T^*(s)u(s) + 2(u(s))^{\frac{1}{2}} \int_{t_0}^{s} [| R'_{1s}(s,\tau) | (T(s))^{-\frac{1}{2}} +$$

$$+ | R'_{2\tau}(s,\tau) | (T(\tau))^{-\frac{1}{2}}](u(\tau))^{\frac{1}{2}} d\tau\} ds, \tag{7}$$

where

$$c_* = (x(t))^2 + \int_{t_0}^{\infty} (f(s))^2 (a(s))^{-1} ds < \infty.$$

Applying Lemma 1 [7] to the integral inequality (7) and taking into account the conditions 2), 3) of the theorem gives the relation $u(t) = O(1).$ From here, based on the inequality $(x(t))^2 \leq u(t)$, it follows that any solution of the IDE (1) $x(t) = O(1).$

Remark. In the theorem there are some number $\varepsilon = const > 0$ and cutting function $\psi(t)$. By selecting them specifically, it is possible to obtain from this theorem the coefficient criterions of the boundednes of all solutions of IDE (1).

Example. For IDE

$$x'(t) + x(t)\sin^2 t + \int_0^t [-\frac{e^{\sqrt{t}\sin t}\sin\tau}{(3t+\tau+2)^2} + \frac{e^{\sqrt{t}\sin t + \sqrt{\tau}\sin\tau}}{\sqrt{1+(t-\tau)e^{-13t}}}] x(\tau) d\tau = -\frac{|\sin t|}{t+3}, \quad t \geq 0$$

all the conditions of the theorem are performed under $\psi(t) \equiv e^{\sqrt{t}\sin t}$, $\varepsilon = \frac{1}{2}$, here

$$t_0 = 0, a(t) \equiv \sin^2 t,\ R_1(t,\tau) \equiv -\frac{\sin\tau}{(3t+\tau+2)^2}, R_2(t,\tau) \equiv \frac{e^{\sqrt{t}\sin t}}{\sqrt{1+(t-\tau)e^{-13t}}},$$

$$T_1(t) \equiv -\frac{e^{-\sqrt{t}\sin t}\sin t}{4(2t+1)^2}, T_2(t) \equiv 1, T(t) \equiv \frac{1}{2} + \frac{e^{-\sqrt{t}\sin t}\sin t}{4(2t+1)^2} > 0, T^*(t) \equiv \frac{\sqrt{t}}{(2t+1)^2} \in L^1(R_+, R_+).$$

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SPECIAL SOLUTIONS OF SYSTEMS OF NON-LINEAR DELAY-DIFFERENTIAL EQUATIONS

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Supra, the author proved existence of “special“ (slowly varying) solutions of initial value problems for scalar non-linear delay-differential equations.
dynamical systems: scalar differential equations with small delay of argument and corresponding difference equations. In the paper existence of “special“ solutions is proven for 2-vector differential equations with small delay of argument and corresponding systems of difference equations is proven.

Key words: delay-differential equation, non-linear equation, special solution, difference equation, initial value problem.

Мурда автор төмөнкүдөй динамикалык системалар үчүн баштапкы маселелердин «атайын» чыгарылыштарынын жашоосун далилдеди: скалярдык, аргументинин кечигүүсү кичине болгон дифференциалдык теңдемелер менен дал келген айырма теңдемелери. Бул макалада «атайын» чыгарылыштарынын жашоосу 2-вектордук аргументинин кечигүүсү кичине болгон дифференциалдык теңдемелер менен дал келген айырма теңдемелери системалары үчүн далилденди.

Урунттуу сөздөр: кечигүүчү аргументтүү дифференциалдык теңдеме, сызыктуу эмес теңдеме, атайын чыгарылыш, айырмалык теңдеме, баштапкы маселе.

Ранее автор доказала существование «специальных» решений начальных задач для динамических систем: скалярных дифференциальных уравнений с малым запаздыванием аргумента и соответствующих разностных уравнений. В статье доказано существование «специальных» решений для 2-векторных дифференциальных уравнений с малым запаздыванием аргумента и соответствующих систем разностных уравнений.

Ключевые слова: дифференциальное уравнение с запаздывающим аргументом, нелинейное уравнение, специальное решение, разностное уравнение, начальная задача.

1. Introduction

Formerly, in order to conduct the in-depth study of differential equations with delay, the author proposed the method of splitting the solution space reducing such equations to the systems of operator-difference equations. Using this method, the author assumed new conditions, i.e. the absolute domains for coefficients sufficient for the existence of special (slowly changing) solutions, and proved the presence of approximating and asymptotically approximating properties in them, as well as the asymptotic one-dimensional space of solutions of the initial problems for linear scalar differential equations with small delay of argument and the corresponding difference equation systems (special solutions correspond, to the solutions with a slowly changing first component and a relatively small second component). For the purposes of the single-point representation of the obtained results and other data related to the theory of dynamic systems (the distance between the solution values tends to zero alongside the unlimited increase in argument), throughout this research paper the author uses the concept of the asymptotic equivalence of solutions for dynamic systems, as it was introduced by the author in their previous papers.

This paper proves existence of “special“ solutions is proven for non-linear vector differential equations with small delay of argument and corresponding systems of difference inequalities.

In order to provide solution of the initial problem for a linear differential equation with a limited retarded argument, some sources, including the book [1] and research papers [2] and [3], discuss the conditions when there exists such one-dimensional sub-space of solutions (referred to as special), so that any solution tends to increase its argument towards one of the special solutions. The review of such academic sources is given in the research paper [4].

The book [5], it is illustrated that using the differential equations with small argument retardation and the technique of splitting the solution space, it is possible to enable their transformation into the systems of operator-difference equations with preservation of their specifics.

Section 2 introduces the required definitions and denotations.

Section 3 presents theorems on existence of special solutions of systems of difference inequalities.

Section 4 presents the newly found conditions for vector differential equations with retarded argument.

2. Denotatons and definitions

Let us denote as follows: $\boldsymbol{N}_0 = \{0,1,2,3,\ldots\}$, $\boldsymbol{N}=\{1,2,3,\ldots\}$, $\boldsymbol{R}=(-\infty,\infty)$, $\boldsymbol{R}_+=[0,\infty)$,$\boldsymbol{R}_{++}=(0,\infty)$, $C^{m(k)}D$ is the space of functions $u\colon D\to\boldsymbol{R}^m$ continuous with the derivatives till the order k;

$||\{u_1,u_2,\ldots u_q\}||_1=||u_1||+||u_2||+\ldots+||u_q||$ is the norm in $\boldsymbol{R}^q$,

D is the domain in $\boldsymbol{R}^q$, $0\in D$, $m\in\boldsymbol{N}$, $k\in\boldsymbol{N}_0$;

$C^{*m(k)}D$ is the sub-space of functions meeting the condition $u(0)=0\in\boldsymbol{R}^m$.

For the purposes of the uniform representation of problems with continuous and discrete time, it shall be assumed that the argument of the desired function t belongs to quite an ordered set Λ, possessing its smallest element (herein referred to as 0), but still not possessing any largest element. Normally, $\Lambda=\boldsymbol{R}_+$ or $\Lambda=\boldsymbol{N}_0$ are employed.

This paper discusses the initial problems only. Assuming that the initial problem always has some solution, then the solution is the only and global one, and, thus, it covers the entire set Λ; so, then the space of solutions of any particular dynamic system with the initial condition φ may be written as the operator $W(t,\varphi)\colon\Lambda\times\Phi\to Z$, with Z being the topological space of the solution values. In the case of $\Lambda=\boldsymbol{R}_+$, let us assume that $W(t,\varphi)$ is continuous with respect to t.

D e f i n i t i o n 2. The following relation of equivalence to space Φ is said to be asymptotic equivalence: If Z is a Banach space, then

$$(\varphi_1\sim\varphi_2)\Leftrightarrow(\lim\{\,||W(t,\varphi_1)-W(t,\varphi_2)||_Z\colon t\to\infty\}=0).$$

D e f i n i t i o n 3. Solutions to $W(t,\varphi)$, $\varphi\in\Phi_0$, are referred to as *special* solutions, if

S1) $(\forall t_1\in\Lambda)\ (\forall z_1\in Z)\ (\exists\varphi_1\in\Phi_0)(W(t_1,\varphi_1)=z_1)$;

S2) $(\exists\lambda > 0)\big(\forall\varphi_1,\varphi_2 \in \Phi_0\big)(\forall t_1, t_2 \in \Lambda)$

$$\Big(\|W(t_1,\varphi_1) - W(t_1,\varphi_2)\| \geq \|W(t_2,\varphi_1) - W(t_2,\varphi_2)\| exp(-\lambda|t_1 - t_2|)\Big).$$

3. System of difference inequalities

Consider the system of difference inequalities

$$x_{n+1} \geq a\,x_n - by_n,\;\; y_{n+1} \leq c\,x_n + d\,y_n,\; n \in \boldsymbol{N}_0, \tag{1}$$

$a, b, c, d \in \boldsymbol{R}_{++}$, x_n , $y_n \in \boldsymbol{R}_+$.

T h e o r e m 1. If (W) $d<a$; $(a - d)^2 > 4bc$ then for numbers

$$w = \Big((a-d) - \sqrt{(a-d)^2 - 4bc}\Big)/(2b),$$

$$q_- = \Big(a + d + \sqrt{(a-d)^2 - 4bc}\Big)/2$$

the following take place: $q_- := a - wb$; $c + wd = wq_-$.

P r o o f by immediate calculation.

T h e o r e m 2. If (W) then a sequence $\{x_n, y_n\}$ fulfilling (1) and $x_0 > 0$, $y_0 = 0$ fulfils $x_n \geq q_-{}^n x_0$, $y_0 \leq wx_n$, $n \in \boldsymbol{N}_0$.

P r o o f by induction. If $x_n \geq q_-{}^n x_0$, $y_0 \leq wx_n$ then

$x_{n+1} \geq a\,x_n - bwx_n = q_- x_n$; $y_{n+1} \leq c\,x_n + dw\,x_n = wq_- x_n \leq w\,x_{n+1}$.

4. Vector non-linear delay-differential equation

Let us consider an equation:

$$w'(t) = P(t, w(t-h))\ (h \in \boldsymbol{R}_{++}),\ t \in \boldsymbol{R}_+, \quad P(t,0) \equiv 0, \tag{2}$$

where $w(t) = \{w_1(t), \ldots, w_q(t)\} \in C^{q(0)}([-h, \infty))$ is the unknown vector-function, and $P(t,w) = \{P_1(t, w), \ldots, P_q(t,w)\} \in C^{q(0)}(\boldsymbol{R}_+)$ is the given vector-function respectively, with the initial condition:

$$w(t) = \varphi(t),\ t \in [-h, 0], \tag{3}$$

where $\varphi(t) = \{\varphi_1(t), \ldots, \varphi_q(t)\} \in C^{q(0)}([-h, 0])$ is the given vector-function;

$$||P(t,w_1) - P(t,w_2)||_1 \leq L\ ||\,w_1 - w_2||_1. \tag{4}$$

It is well-known that the initial problem (2)-(3) is equivalent to the following system of equations for functions $U_n(t) \in C^{q(1)}[-h,0]$, $n \in \boldsymbol{N}_0$:

$$U_0(t) = S_0\big(\varphi(\cdot)\big)(t) := \varphi(0) + \int_{-h}^{t} P(s + h, \varphi(s))\, ds;$$

$$U_{n+1}(t) = S_n\big(U_n(\cdot)\big)(t) := U_n(0) + \int_{-h}^{t} P(s+nh+h, U_n(s))\, ds, n \in \boldsymbol{N}_0. \qquad (5)$$

In [7], the author proposed the following method: let us consider the space $C^{q(1)}[-h,0]=\boldsymbol{R}^q \times C*^{q(1)}[-h,0]$ as Cartesian product of space of functions-constants equal to $\boldsymbol{R}^q$, and the space $\Omega=C*^{q(1)}[-h,0]$, namely $U_n(t)=u_n+z_n(t)$.

Denote that $P_n(\tau) = P(\tau + nh + h)$.

Consequently, the shift operators equal to the value of h in (5) shall be given in the following way:

$$u_{n+1}+z_{n+1}(t) = S_n\big(u_n + z_n(\cdot)\big)(t) = u_n + z_n(0) + \int_{-h}^{t} P_n\big(s, u_n + z_n(s)\big)\, ds =$$

$$= u_n + \int_{-h}^{0} P_n\big(s, u_n + z_n(s)\big)\, ds + \int_{0}^{t} P_n\big(s, u_n + z_n(s)\big)\, ds;$$

$$u_{n+1} = u_n + \int_{-h}^{0} P_n\big(s, u_n + z_n(s)\big)\, ds;$$

$$z_{n+1}(t) = \int_{0}^{t} P_n\big(s, u_n + z_n(s)\big)\, ds. \qquad (6)$$

Introduce the following norm in $C*^{2(1)}[-h,0]$: $||z||_*:=\sup\{||z(t)||_1/|t|:-h\le t<0\}$.

Estimate (6):

$$||u_{n+1}||_1 \ge ||u_n||_1 - \int_{-h}^{0} L(\|u_n + z_n(s)\|_1)\, d|s| \ \ge$$

$$\ge ||u_n||_1 - \int_{-h}^{0} L(\|u_n\|_1 + \|z_n\|_* |s|)\, d|s| =$$

$$= ||u_n||_1 - \|u_n\|_1 Lh - L\|z_n\|_* h^2/2 = ||u_n||_1(1-Lh) - L\|z_n\|_* h^2/2;$$

$$\|z_{n+1}\|_* \le L\|u_n + z_n(s)\|_1 \le L\|u_n\|_1 + L\|z_n\|_* h. \qquad (7)$$

T h e o r e m 3. If $\Delta := Lh < 1/4$ then the equation (3) has solutions fulfilling (S2): $W(0)\neq 0$, $||W(nh)||_1 \ge q_-^{\,n}||W(0)||_1$, $n \in N$.

P r o o f. Apply Theorem 2: $a=1-Lh$; $b=Lh^2/2$; $c=L$; $d=Lh$. Conditions (W):

$1-Lh-Lh>0$; $1-Lh-Lh>4\, Lh^2/2 \cdot L$;

$1-2\Delta > 2\Delta$. Theorem is proven.

Conclusion

The assertion (S1) which is obvious in scalar case is to be investigated in vector case.

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ONE CLASS OF FREDHOLM LINEAR INTEGRAL EQUATIONS OF THE FIRST KIND ON A SEMI-AXIS

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In the present article the theorem about uniqueness of the linear integral equations of the first kind, with method of nonnegative quadratic forms and functional analysis methods.

Key words: linear, inteqral equations, first kind, uniqueness.

Бул макалада терс эмес квадраттык формалар усулунун, функционалдык анализдин усулдарынын жардамы менен биринчи түрдөгү сызыктуу интегралдык тендемелердин чечимдеринин жалгыздыгы далилденди.

Урунттуу сөздөр: сызыктуу, интегралдык тендемелер, биринчи түрдөгү, жалгыздык.

В данной работе, с помощью методом неотрицательных квадратичных форм, методам функционального анализа доказывается единственность решений линейных интегральных уравнений первого рода.

Ключевые слова: линейные, интегральные уравнения, первого рода, единственность.

1. Introduction

Inverse and ill-posed problems are currently attracting great interest. The theory and numerical methods for solving inverse and ill-posed problems were studied in [1-32].The notion of correctness in the works of A.N. Tikhonov [1], M.M. Lavrent'ev [1] and V.K. Ivanov [3], different from the classical one, provided a means for studying ill-posed problems and stimulated interest in integral equations with large applied value.

Ill-posed problems also occur in many other branches of mathematics, elementary examples being the Fredholm integral equation of the first kind, analytic continuation of a function, determination of the derivative of a function that is only approximately specified, and a singular linear system of algebraic equations. A further important class concerns inverse problems where it is typically required to determine the coefficients of an equation from a known ledge of certain functional of the solution.

The fundamental results for Fredholm integral equations of the first kind were obtained by M.M. Lavrentiev in [4], [5], where the regularizing operators in the sense of M.M. Lavrentiev. In [6], [7] results were obtained in the field of nonclassical Volterra equations of the first kind and numerical methods for solving nonclassical linear Volterra equations of the first kind, which describe the processes of aging and replacement of elements in developing systems. In [8], for linear Volterra integral equations of the first and the third kind with smooth kernel, the existence of multiparameter family of solutions was proved. In [9], [10], on the basis of the theory of Volterra integral equations of the first kind, various inverse problems were studied.

Modern methods for solving ill-posed problems are gaining more and more popularity. Results of the inverse problem of microtomography in [11] and numerical methods for solving ill-posed problems in [12] were obtained, inverse problems of biology and medicine [13].

In [14], [15] problems of regularization, uniqueness and of solutions for Volterra integral and operator equations of the first kind are studied.

In [16], a new approach was used to study the existence and uniqueness of solutions to scalar integral Fredholm equations of the third kind with multipoint singularities and their systems.In [17], [18], uniqueness theorems were proved and regularization operators in the sense of Lavrent’ev were constructed for systems of linear Volterra and Fredholm integral equations of the third kind.

In [19], [20] problems of uniqueness and stability of solutions for linear Fredholm integral equations of the first kind were investigated. In [21]-[24] based on the basis of the method of integral transformation, uniqueness theorems for the new class of linear integral equations of the first kind uniqueness theorems for the new class the uniqueness of solutions of linear integral equations and the system of the first kind with two independent variables in the in different areas.

The concept of a derivative with respect to an increasing function was introduced by A. Asanov in 2001 in [25] and plays a special role in the study. This notion is a generalization of the usual notion of a derivative of a function and is an inverse operator for one class of the Stieltjes integral. Based on the concept of such a derivative, using the method of integral transformations and the method of non-negative quadratic forms, linear integral Stieltjes equations of the first kind were studied in [26], [27]. In [28], [29] uniqueness theorems for solutions of the Fredholm and Volterra linear integral equations of the first kind on the semi-axis are proved.

In this work, we apply the method of integral transformation to prove uniqueness theorems for the new class of Fredholm linear integral equations of the first kind in the semi-axis. Consider of the Fredholm linear integral equations

$$Ku \equiv \int_{-\infty}^{a} K(t,s)u(s)ds = f(t), \quad t \in (-\infty, a] \tag{1}$$

where

$$\int_{-\infty}^{a}\int_{-\infty}^{a} |K(t,s)|^2 dsdt < \infty,$$

$$K(t,s) = \begin{cases} A(t,s), & -\infty < s \le t \le a, \\ B(t,s), & -\infty < t \le s \le a. \end{cases} \tag{2}$$

$B(t,s)$ and $A(t,s)$ are given functions, respectively on the domains

$$\{(t,s): -\infty < s \le t \le a\}, \ \{(t,s): -\infty < t \le s \le a\}.$$

$u(t) \in L_2(-\infty, a]$, where $L_2(-\infty, a]$- the space of square-summable functions in $(-\infty, a]$.

Equation (1) by virtue of relation (2) can be expressed as

$$\int_{-\infty}^{t} A(t,s)u(s)ds + \int_{t}^{a} B(t,s)u(s)ds = f(t). \tag{3}$$

Taking the multiplication of both sides of the equation (1) with u(t), integrating the results on $a \le t < \infty$, we obtain.

$$\int_{-\infty}^{a}\int_{-\infty}^{t} A(t,s)u(s)u(t)dsdt+\int_{-\infty}^{a}\int_{t}^{a} B(t,s)u(s)u(t)dsdt=\int_{-\infty}^{a} f(t)u(t)dt,$$

$$\int_{-\infty}^{a}\int_{-\infty}^{t} A(t,s)u(s)u(t)dsdt+\int_{-\infty}^{a}\int_{s}^{a} B(s,t)u(t)u(s)dtds=\int_{-\infty}^{a} f(t)u(t)dt. \qquad (4)$$

Integrating by parts and using the Dirichlet formula we have

$$\int_{-\infty}^{a}\int_{s}^{a} B(s,t)u(t)u(s)dtds=\int_{-\infty}^{a}\left[\int_{-\infty}^{t} B(s,t)u(s)dt\right]u(t)dt.$$

Then from (4) we obtain

$$2\int_{-\infty}^{a}\int_{-\infty}^{t} H(t,s)u(s)ds\,u(t)dt=\int_{-\infty}^{a} f(t)u(t)dt \qquad (5)$$

where

$$H(t,s)=\frac{1}{2}\left[A(t,s)+B(s,t)\right],(t,s)\in\{(t,s):-\infty<s\le t\le a\} \qquad (6)$$

$$\int_{-\infty}^{a}\int_{-\infty}^{t} H^2(t,s)dtds<+\infty.$$

We introduce a new function $M(t,s)$ as follows

$$M(t,s)=\begin{cases} H(t,s), & -\infty<s\le t\le a, \\ H(s,t), & -\infty<t\le s\le a. \end{cases} \qquad (7)$$

It's clear that $M(t,s)=M(s,t),\quad (t,s)\in(-\infty,a]\times(-\infty,a]$.

It is easy to verify the validity of the equality

$$\int_{-\infty}^{a}\int_{-\infty}^{a} |M(t,s)|^2 dsdt<+\infty.$$

Then, it is known that

$$M(t,s)=\sum_{i=1}^{\infty}\frac{\varphi_i(t)\varphi_i(s)}{\lambda_i}, \qquad (8)$$

where, λ_i - the characteristic numbers of the matrix kernel $M(t,s)$, which are arranged in ascending order of their modules, $|\lambda_1|\le|\lambda_2|\le\ldots$ and $\phi_1(t),\phi_2(t)\ldots$ the corresponding orthonormal eigen functions.

Theorem. Suppose $M(t,s)$ is a complete kernel and $0<\lambda_1\le\lambda_2\le\dots$. Then the solution of the equation (1) is unique in $L_2(-\infty,a]$.

Proof. Let equation (1) have a nonzero solution

$u(t)\in L_2(-\infty,a]$ for $f(t)\equiv 0$. Then

$$\int_{-\infty}^{a}K(t,s)u(s)ds=f(t)\equiv 0,\qquad for\ all\ \ t\in(-\infty,a].$$

Hence, by virtue of (5) we have

$$2\int_{-\infty}^{a}\int_{-\infty}^{t}H(t,s)u(s)u(t)dsdt=0 \tag{9}$$

Taking into account (7) and (8) from (9) we get

$$\sum_{i=1}^{\infty}\frac{2}{\lambda_i}\int_{-\infty}^{a}\phi_i(t)u(t)\int_{-\infty}^{t}\phi_i(s)u(s)dsdt=0,$$

$$\sum_{i=1}^{\infty}\frac{1}{\lambda_i}\left|\int_{-\infty}^{a}\phi_i(t)u(t)dt\right|^2=0.$$

Then

$$\int_{-\infty}^{a}\phi_i(t)u(t)dt=0,\quad (i=1,2...).$$

Therefore, $u(t)=0,$ *for all* $t\in(-\infty,a]$. The theorem is proved.

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ON THE METHOD OF LYAPUNOV FUNCTIONALS FOR A CLASS OF WEAKLY NONLINEAR FIRST ORDER INTEGRO-DIFFERENTIAL EQUATIONS OF THE VOLTERRA TYPE

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Sufficient conditions are established for the boundedness on the semiaxis of all solutions of a weakly nonlinear Volterra integro-differential equation of the first order by developing the method of Lyapunov functionals. An illustrative example is being constructed.

Keywords: integro-differential equation, weak nonlinearity, boundedness, Lyapunov functional.

Сызыктуу сымал Вольтерра тибиндеги биринчи тартиптеги интегро-дифференциалдык теңдемелердин бардык чыгарылыштарынын жарым окто чектелгендигинин жетиштүү шарттары Ляпуновдун функционалдар методун өнүктүрүү аркылуу табылат. Иллюстративдик мисал тургузулат.

Урунттуу сөздөр: интегро-дифференциалдык теңдеме, сызыктуу сымалдык, чектелгендик, Ляпуновдун фукнционалы.

Устанавливаются достаточные условия ограниченности на полуоси всех решений слабо нелинейного вольтеррова интегро-дифференциального уравнения первого порядка развитием метода функционалов Ляпунова. Строится иллюстративный пример.

Ключевые слова: интегро-дифференциальное уравнение, слабая нелинейность, ограниченность, функционал Ляпунова.

All appearing functions of are continuous at and relations take place at IDE - an integro-differential equation.

All appearing functions of $t_0, (t,\tau), x, y, z$ are continuous at $t \ge t_0, t \ge \tau \ge t_0,\ |x|,|y| < \infty$ and relations take place at $t \ge t_0, t \ge \tau \ge t_0;\ J = [t_0, \infty);$ IDE - integro-differential equation.

Problem. Establish sufficient conditions for boundedness on the half-interval J of all solutions of the following weakly nonlinear Volterra IDE of the first order of the form:

$$x'(t)+a(t)x(t)+\int_{t_0}^{t}[K(t,\tau)+C(t,\tau)]x(\tau)d\tau =$$

$$= f(t)+F(t,x(t),\int_{t_0}^{t}H(t,\tau,x(\tau))d\tau), \quad t\ge t_0 \qquad (1)$$

in the case when the functions satisfy the conditions of weak nonlinearity

in the case when the functions $F(t,x,y), H(t,\tau,x)$ satisfy the conditions of weak nonlinearity:

$$|F(t,x,y)|\le F_0(t)+g(t)|x|+g_1(t)|y|, |H(t,\tau,x)|\le h(t,\tau)|x| \qquad \text{(SN)}$$

with non-negative functions $F_0(t), g(t), g_1(t), h(t,\tau)$.

Condition (SN) guarantees the existence of a solution $x(t)\in C^1(J,R)$ with any initial data $x(t_0)$.

To solve this problem, the method of Lyapunov functionals is developed, following the works [1, pp.73-77; 2], which makes it possible to study the formulated problem for a new class of IDEs, in contrast to previous studies by other methods (see, for example, [1,3] and bibliographies from these monographs).

Suppose [1]:

$$K(t,\tau)=\sum_{i=0}^{n}K_i(t,\tau), \qquad (K)$$

$$f(t)=\sum_{i=0}^{n}f_i(t), \qquad (f)$$

$\psi_i(t)\ (i=1..n)$ - some cutting functions,

$R_i(t,\tau)\equiv K_i(t,\tau)(\psi_i(t)(\psi_i(\tau))^{-1}, E_i(t)\equiv f_i(t)(\psi_i(t))^{-1}\ (i=1..n),$

$$R_i(t,t_0)=A_i(t)+B_i(t)\ (i=1..n), \qquad (R)$$

$c_i(t)\ (i=1..n)$ - some functions;

$$G(t,\tau)=G_1(t,\tau)G_2(t,\tau)+G_3(t,\tau), \qquad (G)$$

where $G(t,\tau)\equiv g_1(t)h(t,\tau);$

similarly [3, p.216] it is required that the integral $\int_t^\infty C(s,t)ds$ is defined for all $t \geq t_0$.

Theorem. 1) conditions *(K), (f), (R), (G)* are satisfied and the integral $\int_t^\infty C(s,t)ds$ defined for everyone $t \geq t_0$; 2) $D(t) \equiv 2a(t) - \int_{t_0}^t |C(t,\tau)|d\tau - -\int_t^\infty |C(s,t)|ds - \int_{t_0}^t (G_2(t,\tau))^2 d\tau \geq 0$; 3) $A_i(t) \geq 0$, $R'_{1\tau}(t,\tau) \geq 0$, there are functions $A_i^*(t) \in L^1(J,R_+)$, $R_i^*(t) \in L^1(J,R_+)$ such that $A_i'(t) \leq A_i^*(t)A_i(t)$, $R''_{1t\tau}(t,\tau) \leq R_i^*(t)R'_{1\tau}(t,\tau)$ $(i=1..n)$; 4) $B_i(t) \geq 0$, $B_i'(t) \leq 0$, there are functions $c_i(t)$ such that $(E_i^{(k)}(t))^2 \leq B_i^{(k)}(t)c_i^{(k)}(t)$ $(i=1..n)$;

5) $\int_{t_0}^t |K_0(t,\tau)|d\tau + |f_0(t)| + F_0(t) + g(t) + \int_{t_0}^t [(G_1(t,\tau))^2 + |G_3(t,\tau)|]d\tau \in L^1(J,R_+)$.

Then any solution $x(t)$ of IDE (1) is bounded on J and $D(t)(x(t))^2 \in L^1(J,R_+)$.

To prove this theorem, a new Lyapunov functional is constructed in the following form:

$$V(t;x) = (x(t))^2 + \int_{t_0}^t D(s)(x(s))^2 ds + \sum_{i=1}^n [R_i(t,t_0)(X_i(t,t_0))^2 - 2E_i(t)X_i(t,t_0) + c_i(t) + \\ + \int_{t_0}^t R'_{i\tau}(t,\tau)(X_i(t,\tau))^2 d\tau + \int_{t_0}^t (\int_t^\infty |C(s,\tau)|ds)(x(\tau))^2 d\tau, \qquad (2)$$

where

$$X_i(t,\tau) \equiv \int_\tau^t \psi_i(s)x(s)ds \ (i=1..n).$$

Also, when estimating $\dfrac{dV(t;x)}{dt}$ and passing to the integral inequality, the well-known relation is used:

$$2G(t,\tau)x(t)x(\tau) \leq (G_2(t,\tau))^2(x(t))^2 + (G_1(t,\tau))^2(x(\tau))^2 + |G_3(t,\tau)||x(t)x(\tau)|.$$

Otherwise, the proof of Theorem is carried out similarly to the proof of Theorem 1.14 [1, pp.73-76].

In case $F_0(t) \equiv g(t) \equiv g_1(t) \equiv h(t,\tau) \equiv 0$, i.e. for a linear IDE of the form (1) Theorem 1.14 [1, pp.73-76] follows from our Theorem.

Let us consider a linear IDE of the first order in the form:

$$x'(t) + a(t)x(t) + \int_{t_0}^{t} [Q_1(t,\tau)Q_2(t,\tau) + Q_3(t,\tau) + C(t,\tau)]x(\tau)d\tau = f_0(t), t \geq t_0, \qquad (3)$$

which following from IDE (1).

For IDE (3) from Theorem yields the following

Corollary. If the integral

$\int_{t_0}^{\infty} C(s,t)ds$ is defined for all $t \geq t_0$;

$$\Delta(t) \equiv 2a(t) - \int_{t_0}^{t} |C(t,\tau)|\, d\tau - \int_{t}^{\infty} |C(s,t)|\, ds - \int_{t_0}^{t} (Q_2(t,\tau))^2 d\tau \geq 0;$$

$$\int_{t_0}^{t} [(Q_1(t,\tau))^2 + |Q_3(t,\tau)|]d\tau + |f_0(t)| \in L^1(J, R_+),$$

then any solution $x(t)$ of the IDE (3) is bounded on J and the relation $\Delta(t)(x(t))^2 \in L^1(J, R_+)$ is true.

In this case, condition (G) is replaced by a condition $K(t,\tau) \equiv Q_1(t,\tau)Q_2(t,\tau) + Q_3(t,\tau)$ and a Lyapunov functional of the form:

$$V(t;x) = (x(t))^2 + \int_{t_0}^{t} \Delta(s)(x(s))^2 ds$$

is constructed.

Example. IDE

$$x'(t) + (3t + \sqrt[5]{\sin 2t} + 2)x(t) + \int_0^t [\frac{|\cos(t\tau)|}{(t+\tau+4)^{3/2}} + \sin(e^{-t}\sqrt{\tau}) - \frac{(\sin t)^{1/7}}{(t+\tau+8)^{1,2}}]x(\tau)d\tau =$$

$$= -\frac{1}{t^2+4}, \quad t \geq 0$$

satisfies all conditions of Corollary, here

$$t_0 = 0,\ a(t) \equiv 3t + \sqrt[5]{\sin 2t} + 2,\ Q_1(t,\tau) \equiv \frac{1}{(t+\tau+4)^{3/2}},\ Q_2(t,\tau) \equiv |\cos(t\tau)|,$$

$$Q_3(t,\tau) \equiv \sin(e^{-t}\sqrt{\tau}),\ C(t,\tau) \equiv -\frac{(\sin t)^{1/7}}{(t+\tau+8)^{1,2}},\ f_0(t) \equiv -\frac{1}{t^2+4}.$$

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ON BOUNDED SOLUTIONS OF INTEGRO-DIFFERENTIAL PARTIAL DIFFERENTIAL EQUATIONS ON INFINITE DOMAIN

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In this paper we study the limitations of solutions of integral-differential equations with partial derivatives on the endless fields. In order to show the limitations of the method of non-negative solutions of quadratic forms.

Key words: limitations, integration by parts formula Dirichle, integration.

Бул макалада экинчи тартиптеги интегро-дифференциалдык теңдемелердин чексиз аймактардагы чыгарылыштарынын чектелгендиги жөнүндөгү маселе каралат. Терс эмес квадраттык формалар ыкмасын колдонуу менен чечимдердин чектелгендиги көрсөтүлдү.

Урунттуу сөздөр: чектелгендик, бөлүктөп интегралдоо, Дирихленин формуласы, интегралдоо.

В этой статье изучается ограниченность решений интегро-дифференциальных уравнений с частными производными на бесконечных областях. Чтобы показать ограниченность решения применяется метод неотрицательных квадратичных форм.

Ключевые слова: ограниченность, интегрируя по частям, формулы Дирихле, интегрирование.

1. Introduction

In [1-3] considered the question of limiting solutions for an infinite domain of integral and integro-differential equations on the half. In [4-7], results were obtained on the boundedness of solutions of integro-differential equations and systems of integro-differential equations in partial derivatives of the second and third orders in infinite domains. Questions of boundedness of quadratic integrability and stability of solutions for differential, integral, and integro-differential equations have been studied in many works, for example, in [8,9].

In this article, applying the method in [10], we prove a theorem of uniqueness and boundedness of solutions of integro-differential equations in partial derivatives of the second order in infinite domains.

The equation

$$u_{tx}(t,x)+a(t,x)u_x(t,x)+b(t,x)u_t(t,x)+c(t,x)u(t,x)+\int_0^t A(t,x,s)u_{xs}(s,x)ds+$$

$$+\int_0^x B(t,x,y)u_{ty}(t,y)dy+\int_0^t\int_0^x K(t,x,s,y)u_{sy}(s,y)dyds=f(t,x),\ (t,x)\in G\ , \tag{1}$$

$$f(t,x)\in L_2(G)\cap C(G) \tag{f}$$

with conditions

$$u(0,x)=0, \qquad x\in[0,\ +\infty),$$

$$u(t,0)=0, \qquad t\in[0,\ +\infty), \tag{*}$$

where

$A(t,x,s), B(t,x,y), K(t,x,s,y), c(t,x), a(t,x), b(t,x), f(t,x)$ – known features,

a $u(t,x)$ – unknown function

Theorem. If the following conditions are met: (f),

a) functions

$a(t,x),\ b(t,x),\ c(t,x),\ a_t'(t,x),\ b_x'(t,x),\ c_t'(t,x),\ c_x'(t,x),\ c_{tx}''(t,x)$ $\in$ C(G);

$a(t,x)\ge 0,\ a_t'(t,x)\le 0,\quad b(t,x)\ge 0,\quad b_x'(t,x)\le 0$ at

(t,x) $\in$ G;

$c_t'(t,x)\le 0,\quad c_x'(t,x)\le 0,\quad c(t,x)\ge\alpha>0,\quad c_{tx}''(t,x)\ge 0$, $c^2(t,x)-a_t'(t,x)b_x'(t,x)\le 0$

at (t,x) $\in$ G;

б) functions $A(t,x,s), A_t'(t,x,s), A_s'(t,x,s), A_{ts}''(t,x,s)\in C(G_1)$,

$A(t,x,0)\ge 0, A_t'(t,x,0)\le 0$ at $(t,x)\in G$ и $A_s'(t,x,s)\ge 0$ $\quad A_{ts}''(t,x,s)\le 0$ at $(t,x,s)\in G_1$;

в) functions $B(t,x,y), B_x'(t,x,y), B_y'(t,x,y)$ и $B_{xy}''(t,x,y)$ $\in C(G_2)$,

$B(t,x,0)\ge 0, B_x'(t,x,0)\le 0$ at $(t,x)\in G$ и $B_y'(t,x,y)\ge 0$, $B_{xy}''(t,x,y)\le 0$ at $(t,x,y)\in G_2$;

г) functions

$K(t,x,s,y), K'_y(t,x,s,y), K'_s(t,x,s,y), K'_t(t,x,s,y), K'_x(t,x,s,y), K''_{ts}(t,x,s,y)$,

$K''_{tx}(t,x,s,y), K''_{xs}(t,x,s,y), K''_{xy}(t,x,s,y)$, $K'''_{txy}(t,x,s,y), K'''_{tys}(t,x,s,y), K'''_{xsy}(t,x,s,y)$

и $K^{(IV)}_{txsy}(t,x,s,y) \in C(G_3)$,

$K'_y(t,x,0,y) \equiv 0$ at $(t,x,y) \in G_2$, $K'_s(t,x,s,0) \equiv 0$ at $(t,x,s) \in G_1$, $K(t,x,0,0) \geq 0$,

$K'_t(t,x,0,0) \leq 0$, $K'_x(t,x,0,0) \leq 0$, $K''_{tx}(t,x,0,0) \geq 0$ at $(t,x) \in G$ и $K''_{sy}(t,x,s,y) \geq 0$,

$K'''_{tsy}(t,x,s,y) \leq 0,\ K'''_{xsy}(t,x,s,y) \leq 0,\ K^{(IV)}_{txsy}(t,x,s,y) \geq 0$ при $(t,x,s,y) \in G_3$;

д) $K^2(t,x,0,0) - A'_t(t,x,0)B'_x(t,x,0) \leq 0$ at $(t,x) \in G$,

$tx\left(K''_{sy}(t,x,s,y)\right)^2 - A''_{ts}(t,x,s)B''_{xy}(t,x,y) \leq 0$ at $(t,x,s,y) \in G_3$,

то задача (1)–(*) имеет единственное решение в $L_2(G) \cap C(G)$.

Proof. Let's make the following substitution

$$u(t,x) = \int_0^t \int_0^x \vartheta(s,y)\,dyds. \tag{2}$$

Substituting (2) into (1), we have

$$\vartheta(t,x) + a(t,x)\int_0^t \vartheta(s,x)ds + b(t,x)\int_0^t \vartheta(t,y)dy + c(t,x)\int_0^t\int_0^x \vartheta(s,y)dsdy +$$

$$\int_0^t A(t,x,s)\vartheta(s,x)ds + \int_0^x B(t,x,y)\vartheta(t,y)dy + \int_0^t\int_0^x K(t,x,s,y)\vartheta(s,y)dyds = f(t,x). \tag{3}$$

Obviously, problem (1)–(*) is equivalent to the system of integral equations (2)–(3). Both parts of equation (3) multiplied by $\vartheta(t,x)$ and integrating over the region $G_{tx} = \{(s,y): 0 \leq s \leq t,\quad 0 \leq y \leq x\}$, we get

$$\int_0^t\int_0^x \vartheta^2(s,y)dyds + \int_0^t\int_0^x\int_0^s a(s,y)\vartheta(\tau,y)\vartheta(s,y)d\tau dyds + \int_0^t\int_0^x\int_0^y b(s,y)\vartheta(s,z)\vartheta(s,y)dzdyds +$$

$$+\int_0^t\int_0^x\int_0^s\int_0^y c(s,y)\vartheta(\tau,z)\vartheta(s,y)dzd\tau dyds + \int_0^t\int_0^x\int_0^s A(s,y,\tau)\vartheta(\tau,y)\vartheta(s,y)d\tau dyds + \int_0^t\int_0^x\int_0^y B(s,y,z)\times$$

$$\times\vartheta(s,z)\vartheta(s,y)\,dzdyds + \int_0^t\int_0^x\int_0^s\int_0^y K(s,y,\tau,z)\vartheta(\tau,z)\vartheta(s,y)\,dzd\tau dyds = \int_0^t\int_0^x f(s,y)\vartheta(s,y)\,dyds. \tag{4}$$

We transform each of the integrals on the left side of equality (4), with the exception of the first one, according to the integration-by-parts formula and use the following formula

$$\mathrm{KWW_s}' = \frac{1}{2}(KW^2)_s' - \frac{1}{2}K_s'W^2 .$$

$$\int_0^t\int_0^x\int_0^s a(s,y)\vartheta(\tau,y)\vartheta(s,y)d\tau dyds = \int_0^t\int_0^x a(s,y)\left[\int_0^s \vartheta(\tau,y)d\tau\right]\left[\int_0^s \vartheta(\tau,y)d\tau\right]_S' dyds =$$

$$= \frac{1}{2}\int_0^x a\ (t,y)\left(\int_0^t \vartheta(\tau,x)d\tau\right)^2 dy - \frac{1}{2}\int_0^t a_s'(s,y)\left(\int_0^s \vartheta(\tau,y)d\tau\right)^2 dyds.$$

$$\int_0^t\int_0^x\int_0^y b(s,y)\vartheta(s,z)\vartheta(s,y)dzdyds = \int_0^t\int_0^x b(s,y)\left[\int_0^y \vartheta(s,z)dz\right]\left[\int_0^y \vartheta(s,z)dz\right]_y' dyds =$$

$$+\frac{1}{2}\int_0^t b(s,x)\left(\int_0^x \vartheta(s,z)dz\right)^2 - \frac{1}{2}\int_0^t\int_0^x b_y'(s,y)\left(\int_0^y \vartheta(s,z)dz\right)^2 dyds.$$

Next, using the formula $Cvv_{sy}'' = \frac{1}{2}\left(Cv^2\right)_{sy}'' - \frac{1}{2}\left(C_s'v^2\right)_y' - \frac{1}{2}\left(C_y'v^2\right)_s' + \frac{1}{2}C''_{sy}v^2 - Cv_y'v_s'$

$$\int_0^t\int_0^x\int_0^s\int_0^y c(s,y)\vartheta(\tau,z)\vartheta(s,y)dzd\tau dyds = \int_0^t\int_0^x c(s,y)\left[\int_0^s\int_0^y \vartheta(\tau,z)dzd\tau\right]\left[\int_0^s\int_0^y \vartheta(\tau,z)dzd\tau\right]_{sy}'' dyds =$$

$$= \frac{1}{2}c(t,x)\left[\int_0^t\int_0^x \vartheta(\tau,z)dzd\tau\right]^2 - \frac{1}{2}\int_0^t c_s'(s,x)\left(\int_0^s\int_0^x \vartheta(\tau,z)dzd\tau\right)^2 ds - \frac{1}{2}\int_0^x c_y'(t,y)\left(\int_0^t\int_0^y \vartheta(\tau,z)dzd\tau\right)^2 dy +$$

$$+\frac{1}{2}\int_0^t\int_0^x c_{sy}''(s,y)\left(\int_0^s\int_0^y \vartheta(\tau,z)dzd\tau\right)^2 dyds - \int_0^t\int_0^x c(s,y)\left[\int_0^y \vartheta(s,z)dz\right]\left[\int_0^S \vartheta(\tau,y)d\tau\right]dyds. \tag{5}$$

Integrating by parts using the formula

$\mathrm{KWW_s}' = \frac{1}{2}(KW^2)_s' - \frac{1}{2}K_s'W^2$ and the Dirichlet formula, we transform the left side of relation (4)

$$\int_0^t\int_0^x\int_0^s A(s,y,\tau)\vartheta(\tau,y)\vartheta(s,y)d\tau dyds = -\int_0^t\int_0^x\int_0^s A(s,y,\tau)\frac{\partial}{\partial\tau}\left(\int_\tau^s \vartheta(\xi,y)d\xi\right)d\tau\ \vartheta(s,y)dyds =$$

$$= \int_0^t\int_0^x A(s,y,0)\left(\int_0^s \vartheta(\xi,y)d\xi\right)\vartheta(s,y)dyds + \int_0^t\int_0^x\int_0^s A_\tau'(s,y,\tau)\left(\int_\tau^s \vartheta(\xi,y)d\xi\right)\vartheta(s,y)d\tau dyds =$$

$$=\frac{1}{2}\int_0^t A(t,y,0)\left(\int_0^t \vartheta(\xi,y)d\xi\right)^2 dy-\frac{1}{2}\int_0^t\int_0^x A_s'(s,y,0)\left(\int_0^s \vartheta(\xi,y)d\xi\right)^2 dyds+$$

$$+\frac{1}{2}\int_0^t\int_0^x A_\tau'(t,y,\tau)\left(\int_\tau^t \vartheta(\xi,y)d\xi\right)^2 dyd\tau-\frac{1}{2}\int_0^t\int_0^x\int_0^s A_{\tau s}''(s,y,\tau)\left(\int_\tau^s \vartheta(\xi,y)d\xi\right)^2 d\tau dyds. \tag{6}$$

Similarly, we get

$$\int_0^t\int_0^x\int_0^y B(s,y,z)\vartheta(s,z)\vartheta(s,y)dzdyds=\frac{1}{2}\int_0^t B(s,x,0)\left(\int_0^x \vartheta(s,\nu)d\nu\right)^2 ds-\frac{1}{2}\int_0^t\int_0^x B_y'(s,y,0)\left(\int_0^y \vartheta(s,\nu)d\nu\right)^2 dyds+$$

$$+\frac{1}{2}\int_0^t\int_0^x B_z'(s,x,z)\left(\int_z^x \vartheta(s,\nu)d\nu\right)^2 dzds-\frac{1}{2}\int_0^t\int_0^x\int_0^y B_{zy}''(s,y,z)\left(\int_z^y \vartheta(s,\nu)d\nu\right)^2 dzdyds. \tag{7}$$

To transform the integral, we use the following formula

$$C\nu_{\tau z}''=(C\nu)''_{\tau z}-(C_\tau'\nu)'_z-(C_z'\nu)'_\tau+C''_{\tau z}\nu$$

at $\nu(\mathrm{s,y},\tau,\mathrm{z})=\int_\tau^s\int_z^y \vartheta_1(\xi,\nu)d\nu d\xi$.

We transform the last integral on the left side of relation (4):

$$\int_0^t\int_0^x\int_0^s\int_0^y K(s,y,\tau,z)\vartheta(\tau,z)\vartheta(s,y)dzd\tau dyds=$$

$$=\int_0^t\int_0^x\left[\int_0^s\int_0^y \frac{\partial^2}{\partial\tau\partial z}\left(K(s,y,\tau,z)\int_\tau^s\int_z^y \vartheta(\xi,\nu)d\nu d\xi\right)dzd\tau-\int_0^s\int_0^y \frac{\partial}{\partial z}\left(K_\tau'(s,y,\tau,z)\int_\tau^s\int_z^y \vartheta(\xi,\nu)d\nu d\xi\right)dzd\tau-\right.$$

$$\left.-\int_0^s\int_0^y \frac{\partial}{\partial\tau}\left(K_z'(s,y,\tau,z)\int_\tau^s\int_z^y \vartheta(\xi,\nu)d\nu d\xi\right)dzd\tau+\int_0^s\int_0^y K_{\tau z}''(s,y,\tau,z)\left(\int_\tau^s\int_z^y \vartheta(\xi,\nu)d\nu d\xi\right)dzd\tau\right]\vartheta(s,y)dyds=$$

$$=\int_0^t\int_0^x K(s,y,0,0)\left(\int_0^s\int_0^y \vartheta(\xi,\nu)d\nu\, d\xi\right)\vartheta(s,y)dyds+\int_0^t\int_0^x\int_0^s K_\tau'(s,y,\tau,0)\int_\tau^s\int_0^y \vartheta(\xi,\nu)d\nu\, d\xi\, \vartheta(s,y)d\tau\, dyds+$$

$$+\int_0^t\int_0^x\int_0^y K_z'(s,y,0,z)\int_0^s\int_z^y \vartheta(\xi,\nu)d\nu d\xi\, \vartheta(s,y)dzdyds+\int_0^t\int_0^x\int_0^s\int_0^y K_{\tau z}''(s,y,\tau,z)\times$$

$$\times\left(\int_\tau^s\int_z^y \vartheta(\xi,\nu)d\nu d\xi\right)\vartheta(s,y)dzd\tau dyds. \tag{8}$$

Next, using the formula

$$C\nu\nu_{sy}''=\frac{1}{2}\left(C\nu^2\right)''_{sy}-\frac{1}{2}\left(C_s'\nu^2\right)'_y-\frac{1}{2}\left(C_y'\nu^2\right)'_s+\frac{1}{2}C''_{sy}\nu^2-C\nu_y'\nu_s'$$

and the Dirichlet formula, from the last relation, we obtain

$$\int_0^t\int_0^x\int_0^s\int_0^y K(s,y,\tau,z)u(\tau,z)u(s,y)dzd\tau dyds=\frac{1}{2}K(t,x,0,0)\left(\int_0^t\int_0^x u(\xi,\nu)d\nu d\xi\right)^2-$$

$$-\frac{1}{2}\int_0^t K'_s(s,x,0,0)\left(\int_0^s\int_0^x \vartheta(\xi,\nu)d\nu d\xi\right)^2 ds-\frac{1}{2}\int_0^x K'_y(t,y,0,0)\left(\int_0^t\int_0^y \vartheta(\xi,\nu)d\nu d\xi\right)^2 dy+$$

$$+\frac{1}{2}\int_0^t\int_0^x K''_{sy}(s,y,0,0)\left(\int_0^s\int_0^y \vartheta(\xi,\nu)d\nu d\xi\right)^2 dyds-\int_0^t\int_0^x K(s,y,0,0)\left(\int_0^s \vartheta(\xi,y)d\xi\right)\left(\int_0^y \vartheta(s,\nu)d\nu\right)dyds+$$

$$+\frac{1}{2}\int_0^t K'_\tau(t,x,\tau,0)\left(\int_\tau^t\int_0^x \vartheta(\xi,\nu)d\xi d\nu\right)^2 d\tau-\frac{1}{2}\int_0^t\int_0^s K''_{\tau s}(s,x,\tau,0)\left(\int_\tau^s\int_0^x \vartheta(\xi,\nu)d\nu d\xi\right)^2 d\tau ds-$$

$$-\frac{1}{2}\int_0^t\int_0^x K''_{\tau y}(t,y,\tau,0)\left(\int_\tau^t\int_0^y \vartheta(\xi,\nu)d\nu d\xi\right)^2 dyd\tau+\frac{1}{2}\int_0^t\int_0^x\int_0^s K'''_{\tau sy}(s,y,\tau,0)\left(\int_\tau^s\int_o^y \vartheta(\xi,\nu)d\nu d\xi\right)^2 d\tau dyds-$$

$$-\int_0^t\int_0^x\int_0^s K'_\tau(s,y,\tau,0)\left(\int_\tau^s \vartheta(\xi,y)d\xi\right)\left(\int_0^y \vartheta(s,\nu)d\nu\right)d\tau dyds+\frac{1}{2}\int_0^x K'_z(t,x,0,z)\left(\int_0^t\int_z^x \vartheta(\xi,\nu)d\nu d\xi\right)^2 dz-$$

$$-\frac{1}{2}\int_0^t\int_0^x K''_{zs}(s,x,0,z)\left(\int_0^s\int_z^x \vartheta(\xi,\nu)d\nu d\xi\right)^2 dzds-\frac{1}{2}\int_0^x\int_0^y K''_{zy}(t,y,o,z)\left(\int_0^t\int_z^y \vartheta(\xi,\nu)d\nu d\xi\right)^2 dzdy+$$

$$+\frac{1}{2}\int_0^t\int_0^x\int_0^y K'''_{zsy}(s,y,0,z)\left(\int_0^s\int_z^y \vartheta(\xi,\nu)d\nu d\xi\right)^2 dzdyds-\int_0^t\int_0^x\int_0^y K'_z(s,y,0,z)\left(\int_0^s \vartheta(\xi,y)d\xi\right)\times$$

$$\times\left(\int_z^y \vartheta(s,\nu)d\nu\right)dzdyds+\frac{1}{2}\int_0^t\int_0^x K''_{\tau z}(t,x,\tau,z)\left(\int_\tau^t\int_z^x \vartheta(\xi,\nu)d\nu d\xi\right)^2 dzd\tau-\frac{1}{2}\int_0^t\int_0^x\int_0^y K'''_{\tau zy}(t,y,\tau,z)\times$$

$$\times\left(\int_\tau^t\int_z^y \vartheta(\xi,\nu)d\nu d\xi\right)^2 dzdyd\tau-\frac{1}{2}\int_0^t\int_0^x\int_0^s K'''_{\tau zs}(s,x,\tau,z)\left(\int_\tau^s\int_z^x \vartheta(\xi,\nu)d\nu d\xi\right)^2 d\tau dzds-$$

$$-\int_0^t\int_0^x\int_0^s\int_0^y K''_{\tau z}(s,y,\tau,z)\left(\int_\tau^s \vartheta(\xi,y)d\xi\right)\left(\int_z^y \vartheta(s,\nu)d\nu\right)dzd\tau dyds+$$

$$+\frac{1}{2}\int_0^t\int_0^x\int_0^s\int_0^y K^{(IV)}_{z\tau sy}(s,y,\tau,z)\left(\int_\tau^s\int_z^y \vartheta(\varsigma,\nu)d\nu d\xi\right)^2 dzd\tau dyds. \qquad (9)$$

Taking into account relations (5), (6), (7), (9), condition d) and the Dirichlet formula, from (4) we have

$$\int_0^t\int_0^x \vartheta^2(s,y)dyds+\frac{1}{2}\left\{\int_0^x a(t,y)\left(\int_0^t \vartheta(\tau,y)d\tau\right)^2 dy+\int_0^t b(s,x)\left(\int_0^x \vartheta(s,z)dz\right)^2 ds-\right.$$

$$-\int_0^t c_s'(s,x)\left(\int_0^s\int_0^x \vartheta(\tau,z)dzd\tau\right)^2 ds-\int_0^x c_y'(t,y)\left(\int_0^t\int_0^y \vartheta(\tau,z)dzd\tau\right)^2 dy\Bigg\}+\frac{1}{2}c(t,x)\left[\int_0^t\int_0^x \vartheta(\tau,z)dzd\tau\right]^2+$$

$$+\frac{1}{2}\int_0^t\int_0^x\Bigg\{-a_s'(s,y)\left(\int_0^s \vartheta(\tau,y)d\tau\right)^2-2c(s,y)\left(\int_0^s \vartheta(\tau,y)d\tau\right)\left(\int_0^y \vartheta(s,z)dz\right)-$$

$$-b_y'(s,y)\left(\int_0^y \vartheta(s,z)dz\right)^2\Bigg\}dyds+\frac{1}{2}\int_0^t\int_0^x\ c_{sy}''(s,y)\left(\int_0^s\int_0^y \vartheta(\tau,z)dzd\tau\right)^2 dyds+$$

$$+\frac{1}{2}\int_0^x A(z,y,0)\left(\int_0^t \vartheta(\xi,y)d\xi\right)^2 dy+\frac{1}{2}\int_0^t B(s,x,0)\left(\int_0^x \vartheta(s,\nu)d\nu\right)^2 ds+$$

$$+\frac{1}{2}\int_0^t\int_0^x B_z'(s,x,z)\left(\int_z^x \vartheta(s,\nu)d\nu\right)^2 dzds+\frac{1}{2}\int_0^t\int_0^x A_\tau'(t,y,\tau)\left(\int_\tau^t \vartheta(\xi,y)d\xi\right)^2 dyd\tau-$$

$$-\frac{1}{2}\int_0^t\int_0^x\Bigg\{A_s'(s,y,0)\left(\int_0^s \vartheta(\xi,y)d\xi\right)^2+2K(s,y,0,0)\left(\int_0^s \vartheta(\xi,y)d\xi\right)\left(\int_0^y \vartheta(s,\nu)d\nu\right)+$$

$$+B_y'(s,y,0)\left(\int_0^y \vartheta(s,\nu)d\nu\right)^2\Bigg\}dyds-\frac{1}{2}\int_0^t\int_0^x\int_0^s\int_0^y\Bigg\{\frac{1}{y}A_{\tau s}''(s,y,\tau)\left(\int_\tau^s \vartheta(\xi,y)d\xi\right)^2+$$

$$+2K_{\tau z}''(s,y,\tau,z)\left(\int_\tau^s \vartheta(\xi,y)d\xi\right)\left(\int_z^y \vartheta(s,\nu)d\nu\right)+\frac{1}{s}B_{zy}''(s,y,z)\left(\int_z^y \vartheta(s,\nu)d\nu\right)^2\Bigg\}dzd\tau dyds+$$

$$+\frac{1}{2}K(t,x,0,0)\left(\int_0^t\int_0^x \vartheta(\xi,\nu)d\nu d\xi\right)^2-\frac{1}{2}\int_0^t K_s'(s,x,0,0)\left(\int_0^s\int_0^x \vartheta(\xi,\nu)d\nu d\xi\right)^2 ds-$$

$$-\frac{1}{2}\int_0^x K_y'(t.y,0,0)\left(\int_0^t\int_0^y \vartheta(\xi,\nu)d\nu d\xi\right)^2 dy+\frac{1}{2}\int_0^t\int_0^x K_{sy}''(s,y,0,0)\left(\int_0^s\int_0^y \vartheta(\xi,\nu)d\nu d\xi\right)^2 dyds+$$

$$+\frac{1}{2}\int_0^t\int_0^x K_{\tau z}''(t,x,\tau,z)\left(\int_\tau^t\int_z^x \vartheta(\xi,\nu)d\nu d\xi\right)^2 dzd\tau-\frac{1}{2}\int_0^t\int_0^x\int_0^y K_{\tau zy}'''(t,y,\tau,z)\left(\int_\tau^t\int_z^y \vartheta(\xi,\nu)d\nu d\xi\right)^2 dzdyd\tau-$$

$$-\frac{1}{2}\int_0^t\int_0^x\int_0^s K_{\tau zs}'''(s,x,\tau,z)\left(\int_\tau^s\int_z^x \vartheta(\xi,\nu)d\nu d\xi\right)^2 d\tau dzds+\frac{1}{2}\int_0^t\int_0^x\int_0^s\int_0^y K_{z\tau sy}^{(IV)}(s,y,\tau,z)\times$$

$$\times\left(\int_{\tau}^{s}\int_{z}^{y}\vartheta(\xi,\nu)d\nu d\xi\right)^{2} dzd\tau dyds=\int_{0}^{t}\int_{0}^{x} f(s,y)\vartheta(s,y)dyds. \qquad (10)$$

By virtue of conditions a) – e), the left side of relation (10) is non-negative, so this implies the following inequality

$$\int_{0}^{t}\int_{0}^{x}\vartheta^{2}(s,y)dyds+\frac{\alpha}{2}\left(\int_{0}^{t}\int_{0}^{x}\vartheta(s,y)dyds\right)^{2}\le\int_{0}^{t}\int_{0}^{x} f(s,y)\vartheta(s,y)dyds. \qquad (11)$$

Then, by substitution (2), this is equivalent to the inequality

$$\int_{0}^{t}\int_{0}^{x}\vartheta^{2}(s,y)dyds+\frac{\alpha}{2}u^{2}(t,x)\le\left|\int_{0}^{t}\int_{0}^{x} f(s,y)\vartheta(s,y)dyds\right|. \qquad (12)$$

we have further

$$\int_{0}^{t}\int_{0}^{x} f(s,y)\vartheta(s,y)dyds\le\frac{1}{2}\int_{0}^{t}\int_{0}^{x} f^{2}(s,y)dyds+\frac{1}{2}\int_{0}^{t}\int_{0}^{s}\vartheta^{2}(s,y)dyds.$$

With this in mind, from (12) we have

$$\int_{0}^{t}\int_{0}^{x}\vartheta^{2}(s,y)dyds+\alpha u^{2}(t,x)\le\int_{0}^{t}\int_{0}^{x} f^{2}(s,y)dyds.$$

Hence, by virtue of the condition, (f) we have

$$\int_{0}^{t}\int_{0}^{x}\vartheta^{2}(s,y)dyds+\alpha u^{2}(t,x)\le\int_{0}^{\infty}\int_{0}^{\infty} f^{2}(s,y)dyds<\infty,$$

from which follows

$$u^{2}(t,x)\le\frac{1}{\alpha}\int_{0}^{\infty}\int_{0}^{\infty} f^{2}(s,y)dyds.$$

Consequently, $u(t,x)$ limited at $(t,x)\in G$.Thus, the theorem is proved.

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SOLUTION OF A PERTURBED NONLINEAR DIFFERENTIAL EQUATION WITH A WEAK SINGULAR POINT

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This article considers a perturbed differential equation with a famous singularity in the case of a rational exponent of a singular point. We study the behavior of the solution of the perturbed equation up to and including the singular point.

Key words: singular point, small parameter, problem solution, unperturbed problem, exponential term, coefficients, asymptotic series, equation, mathematical induction principle, closed ball, contraction mapping principle.

Бул макалада сингулярдуу чекиттин рационалдуу көрсөткүчү болгон учурда сингулярдуу козголгон дифференциалдык теңдеме каралат. Биз сингулярдуу чекитти камтыган жана анын ичинде козголгон теңдеменин чечиминин абалын изилдейбиз.

Урунттуу сөздөр: сингулярдык чекит, кичинекей параметр, маселенин чечими, козголбогон маселе, экспоненциалдык мүчө, коэффициенттер, асимптотикалык катар, теңдеме, математикалык индукция принциби, туюк шар, кысып чагылдыруу принциби.

В данной статье рассматривается возмущенное дифференциальное уравнения со слабой особенностью в случае рационального показателя особой точки. Изучается поведение решения возмущенного уравнения до особой точки включительно.

Ключевые слова: особая точка, малый параметр, решение задачи, невозмущенная задача, экспоненциальный член, коэффициенты, асимптотический ряд, уравнение, принцип математической индукции, замкнутый шар, принцип сжимающих отображений.

Consider the Cauchy problem for the equation

$$Lu_\varepsilon := \left(x^{1/n_0} + \varepsilon u(x)\right) u'(x) = q(x)u(x) + r(x), \quad u(1) = u^0. \tag{1}$$

where $0 < \varepsilon \ll 1$ – small parameter, $n_0 \geq 2$, $u^0 -$ given constant, $x \in [0,1]$. The following conditions are imposed on the known functions:

$$U: \ q(x), \ r(x) \in C^{\infty}[0,1]. \qquad q(x) = \sum_{k=0}^{\infty} q_k x^k, \ \ r(x) = \sum_{k=0}^{\infty} r_k x^k,$$

$$q_k = q^{(k)}(0) \ / \ k!\,, \qquad r_k = r^{(k)}(0) \ / \ k!$$

The following questions arise: 1) Does problem (1) have a solution on the segment [0,1]? 2) If so, is it possible to obtain a uniformly suitable solution over the entire interval [0,1]?

The unperturbed problem ($\varepsilon = 0$), corresponding to problem (1) has the form:

$$Lu_0(x) := x^{1/n_0} u_0'(x) \ - \ q(x) u_0(x) = r(x), \qquad u_0(1) = u^0. \tag{2}$$

For equation (2), the point $x=0$ is a weak singular point. The asymptotics of the solution of the unperturbed problem for $x \to 0$ can be obtained directly from [1]:

$$u_0(x) = w_0 + x^{1-1/n_0} \textstyle\sum_{i+j=0}^{\infty} u_{ij}^{(0)} \, x^{(1-1/n_0)i} x^j, \qquad (x \to 0) \tag{3}$$

where

$$w_0 = w(0) = G(0) \left[u^0 \ - \int_0^1 G^{-1}(s) S^{-1/n_0} r(s) ds \right] \neq 0 \tag{4}$$

$$G(x) = exp\left\{ \int_1^x S^{-1/n_0} \, q(s) ds \right\} = G(0) exp\left\{ \int_0^x S^{-1/n_0} \, q(s) ds \right\} =$$

$$= G(0) \ exp\left\{ x^{1-1/n_0} \sum_{j=0}^{\infty} q_j \ / \ (j + 1 - 1/n_0) x^j \right\} \tag{5}$$

If we provide the exponential term as a series in powers of x, then formula (5) can be written as

$$G(x) = G(0) \left[1 + x^{1-1/n_0} \sum_{i+j=0}^{\infty} a_{ij} \, x^{(1-1/n_0)i} x^j \right],$$

$$G^{-1}(x) = G^{-1}(0) \left[1 + x^{1-1/n_0} \sum_{i+j=0}^{\infty} b_{ij} \, x^{(1-1/n_0)i} x^j \right],$$

where

$$G(0) = exp\left\{ \int_0^0 S^{-1/n_0} q(s)ds \right\}$$

$$g(x) = \sum_{j=0}^{\infty} q_j x^j / (j + 1 - 1 / n_0),$$

a_{ij}, b_{ij} – constant coefficients independent of *x*.

$$u_0'(x) = x^{-1/n_0} \sum_{i+j=0}^{\infty} \bar{u}_{ij}^{(0)} x^{(1-1/n_0)i} \cdot x^j \qquad (x \to 0), \qquad (6)$$

where $u_{ij}^{(0)}, \ \bar{u}_{ij}^{(0)}$ - constant coefficients independent of *x*.

We will look for a solution to problem (1) using the small parameter method, i.e., in the form of a row

$$u(x, \varepsilon) = u_0(x) + \varepsilon u_1(x) + \varepsilon^2 u_2(x) + \cdots + \varepsilon^n u_n(x) + \cdots \qquad (7)$$

Substituting (7) into (1) and equating the coefficients at the same powers ε in both parts of the equality, we arrive at the equations:

$$Lu_1(x) = -\, u_0(x) u_0'(x), \qquad u_1(1) = 0, \qquad (7.1)$$

$$Lu_2(x) = -\, u_0(x) u_1'(x) - u_1(x) u_0'(x), \qquad u_2(1) = 0, \qquad (7.2)$$

$$\cdots$$

$$Lu_n(x) = -\sum_{i+j=n-1} u_i(x) u_j'(x), \qquad u_n(1) = 0. \qquad (7.n)$$

then to determine $u_j(x)$ respectively, we use formulas (7.n).

By virtue of equalities (3) and (6), equation (7.1) can be written as:

$$Lu_1(x) = -\left(w_0 + x^{1-1/n_0} \sum_{i+j=0}^{\infty} u_{ij}^{(0)} x^{(1-1/n_0)i} x^j \right)\left(x^{-1/n_0} \sum_{i+j=0}^{\infty} \bar{u}_{ij}^{(0)} x^{(1-1/n_0)} \cdot x^j \right), \ \ u_1(1) = 0.$$

$$Lu_1(x) = x^{-1/n_0} \sum_{i+j=0}^{\infty} u_{ij2}^{(0)} x^{(1-1/n_0)i} x^j,$$

Integrating the last equality, we have:

$$u_1(x) = G(x) J_1(x), \qquad (8)$$

where

$$J_1(x) \equiv G^{-1}(0) \int_0^x \left(s^{-2/n_0} \sum_{i+j=0}^{\infty} u_{ij2}^{(0)} s^{(1-1/n_0)i} s^j \right) G^{-1}(s)\, ds$$

Putting in (8) the found value $J_1(x)$, we get:

$$u_1(x) = x^{1-2/n_0} \sum_{i+j=0}^{\infty} u_{ij}^{(1)} x^{(1-1/n_0)i} x^j + C_1 + x^{1-1/n_0} \sum_{j=0}^{\infty} u_{j0}^{(1)} x^j, \tag{9}$$

$$u_1'(x) = x^{-2/n_0} \sum_{i+j=0}^{\infty} u_{ij}^{(1)} x^{(1-1/n_0)i} x^j + x^{-1/n_0} \sum_{j=0}^{\infty} u_{j0}^{(1)} x^j. \tag{10}$$

where $C_1 = const.$

Taking into account equalities (3), (6), (9) and (10), equation (7.2) will be rewritten in the following form:

$$Lu_2(x) = -\left(w_0 + x^{1-1/n_0} \sum_{i+j=0}^{\infty} u_{ij}^{(0)} x^{(1-1/n_0)i} x^j \right) \times$$

$$\times \left(x^{-2/n_0} \sum_{i+j=0}^{\infty} \bar{u}_{ij}^{(0)} x^{(1-1/n_0)} \cdot x^j + x^{-1/n_0} \sum_{i+j=0}^{\infty} \bar{u}_{j0}^{(1)} \cdot x^j \right) \times$$

$$\times \left(x^{1-2/n_0} \sum_{i+j=0}^{\infty} u_{ij}^{(1)} x^{(1-1/n_0)i} x^j + C_1 + x^{1-1/n_0} \sum_{i+j=0}^{\infty} u_{j0}^{(1)} x^j \right) -$$

$$-\left(x^{-1/n_0} \sum_{i+j=0}^{\infty} \bar{u}_{ij}^{(0)} x^{(1-1/n_0)} \cdot x^j \right), \qquad u_2(1) = 0.$$

Hence, grouping the coefficients at the same powers *x*, we get:

$$Lu_2(x) = x^{-2/n_0} \sum_{j=0}^{\infty} \tilde{u}_{0j}^{(2)} x^j + x^{1-3/n_0} \sum_{j=0}^{\infty} \tilde{u}_{1j}^{(2)} x^j +$$

$$+ x^{2-4/n_0} \sum_{j=0}^{\infty} \tilde{u}_{2j}^{(2)} x^j + \ldots + x^{-1/n_0} \sum_{j=0}^{\infty} \tilde{u}_{j0}^{(2)} x^j. \tag{11}$$

Integrating the last equality (11), we have:

$$u_2(x) = G(x) J_2(x) \tag{12}$$

where

$$J_2(x) = x^{1-3/n_0} \sum_{i+j=0}^{\infty} \tilde{u}_{ij3}^{(2)} x^{(1-1/n_0)i} x^j + x^{1-2/n_0} \sum_{j=0}^{\infty} \tilde{u}_{j1}^{(2)} x^j + C_2 \qquad (13)$$

where $C_2 = const.$

Putting (13) into formula (12), we get:

$$u_2(x) = x^{1-3/n_0} \sum_{i+j=0}^{\infty} u_{ij}^{(2)} x^{(1-1/n_0)i} x^j + x^{1-2/n_0} \sum_{j=0}^{\infty} u_{j0}^{(2)} x^j + C_2 +$$

$$+x^{1-1/n_0} \sum_{j=0}^{\infty} u_{j1}^{(2)} x^j, \qquad (14)$$

$$u_2'(x) = x^{-3/n_0} \sum_{i+j=0}^{\infty} \bar{u}_{ij}^{(2)} x^{(1-1/n_0)i} x^j + x^{-2/n_0} \sum_{j=0}^{\infty} \bar{u}_{j0}^{(1)} x^j +$$

$$+x^{-1/n_0} \sum_{j=0}^{\infty} \bar{u}_{j0}^{(2)} x^j. \qquad (15)$$

Similarly, by induction from the formula $(7.\, n_0 + k)$ we have:

$$u_{n_0+k}(x) = u_0^{(n_0+k)} x^{-(k+1)/n_0} + u_1^{(n_0+k)} x^{-k/n_0} + u_2^{(n_0+k)} x^{-k/n_0} + \ldots +$$

$$+ P_{n_0+k-2}(lnx) + x^{1-1/n_0} \sum_{i+j=0}^{\infty} u_{ij}^{(n_0+k)} x^{(1-1/n_0)i} x^j,$$

where $P_{n_0+k-2}(lnx) - n_0 + k - 2$ – th degree polynomial in lnx function

or

$$u_{n_{0+k}}(x) \sim 0(x^{-(k+1)/n_0}) \qquad (x \to 0) \qquad (16)$$

Consequently, series (7) will be rewritten in the form:

$$u(x,\varepsilon) = w_0 + x^{1-1/n_0} \sum_{i+j=0}^{\infty} u_{ij}^{(0)} x^{(1-1/n_0)i} x^j + \varepsilon[\, x^{1-2/n_0} \sum_{i+j=0}^{\infty} u_{ij}^{(1)} x^{(1-1/n_0)i} x^j +$$

$$+C_1 + x^{1-1/n_0} \sum_{j=0}^{\infty} u_{j0}^{(1)} x^j] + \varepsilon^2[\, x^{1-3/n_0} \sum_{i+j=0}^{\infty} u_{ij}^{(2)} x^{(1-1/n_0)i} x^j +$$

$$+ x^{1-2/n_0} \sum_{j=0}^{\infty} u_{j0}^{(2)} x^j + C_2 + x^{1-1/n_0} \sum_{j=0}^{\infty} u_{j1}^{(2)} x^j] + \ldots +$$

$$+\varepsilon^{n_0-2}[\,x^{1/n_0}\sum_{i+j=0}^{\infty}u_{ij}^{(n_0-2)}x^{(1-1/n_0)i}\,x^j+\dots+\,x^{1-1/n_0}\sum_{j=0}^{\infty}u_{j(n_0-3)}^{(n_0-2)}\,x^j]+$$

$$+\varepsilon^{n_0-1}[lnx(u_{00}^{(n_0-1)}+\,x^{1-1/n_0}\sum_{i+j=0}^{\infty}u_{ij}^{(n_0-1)}\,x^{(1-1/n_0)i}x^j+\,\dots+$$

$$+\,x^{1-2/n_0}\sum_{j=0}^{\infty}u_j^{(n_0-1)}\,x^j]\,+\varepsilon^{n_0}[u^{(n_0)}x^{-1/n_0}\,lnx(u_{00}^{(n_0)}+\,u_{01}^{(n_0)}x^{1-1/n_0}+$$

$$+x^{1-2/n_0}\sum_{i+j=0}^{\infty}u_{ij}^{(n_0)}\,x^{(1-1/n_0)i}x^j+\,\dots+\,\,x^{1-3/n_0}\sum_{j=0}^{\infty}u_{j(n_0-3)}^{(n_0)}\,x^j]\,+\,\dots+$$

$$+\,\varepsilon^{n_0+k}[u^{(n_0+k)}x^{-(k+1)/n_0}+u_1^{(n_0+k)}x^{-k/n_0}\,+\dots+\,P_{n_0+k-2}(lnx)+$$

$$+\,x^{1-1/n_0}\sum_{i+j=0}^{\infty}u_{ij}^{(n_0+k)}\,x^{(1-1/n_0)i}x^j]+\dots \qquad (17)$$

The resulting series (17), for $x\to 0$ has the form:

$$u(x,\varepsilon)\sim w_0+O(\varepsilon)+O(\varepsilon^2)+\,\dots+O(\varepsilon^{n_0-2})+O(\varepsilon^{n_0-1}\,ln\,x)+$$

$$+\varepsilon^{n_0}x^{-1/n_0}\big[w_1+O\big(\varepsilon x^{-1/n_0}\big)\big]+O\big(\varepsilon x^{-1/n_0}\big)^2+\dots+O\big(\varepsilon x^{-1/n_0}\big)^k+\dots].$$

Consequently, $$\frac{\left(\varepsilon x^{-\frac{1}{n_0}}\right)^k}{\left(\varepsilon x^{-\frac{1}{n_0}}\right)^{k-1}}=\varepsilon x^{-\frac{1}{n_0}}$$

Hence we have:

$$\varepsilon x^{-1/n_0}=\varepsilon^{\beta}, \qquad x_0(\varepsilon)=\varepsilon^{n_0(1-\beta)}, \qquad 0<\beta<1.$$

Series (17) is an asymptotic series on the segment $\left[\varepsilon^{n_0(1-\beta)},1\right]$.

$$u(x_0,(\varepsilon),\varepsilon)\sim w_0+O(\varepsilon)+O(\varepsilon^2)+\,\dots+O(\varepsilon^{n_0-2})+O(\varepsilon^{n_0-1}\,ln\,x\,(1/\varepsilon))+$$

$$+\varepsilon^{n_0-1+\beta}\big[w_1+O\big(\varepsilon^{\beta}\big)+O\big(\varepsilon^{2\beta}\big)+\dots+O\big(\varepsilon^{k\beta}\big)+\dots\big],$$

$$u(x_0,(\varepsilon),\varepsilon)\sim w_0+O(\varepsilon^{n_0-1}\,ln\,x\,(1/\varepsilon)) \qquad (x\to 0) \qquad (18)$$

Thus, the following theorem is proved.

THEOREM 1. Let the following conditions be satisfied: 1) $\alpha=1/n_0,(n_0\geq 2)$, functions $q(x),r(x)$ – expands into an asymptotic series in powers of x on the segment [0,1]; 2) $\lim_{x\to 0}u_0(x)=w_0\neq 0$. Then the solution of problem (1) obtained by the small parameter method is an asymptotic solution on the segment $[\varepsilon^{n_0(1-\beta)},1]$.

To continue the solution to the segment $\left[0,\varepsilon^{n_0(1-\beta)}\right]$ in (1) we make a substitution:

$$x=\varepsilon^{n_0}\tau, \qquad u(x)=w_0+v(\tau) \tag{19}$$

Then we have

$$\left(\varepsilon\tau^{1/n_0}+\varepsilon(w_0)+v(\tau)\right)\varepsilon^{-n_0}dv/d\tau \;=\; q(\varepsilon^{n_0}\tau)\left(w_0+v(\tau)\right)+r(\varepsilon^{n_0}\tau)\,,$$

$$\left(\tau^{1/n_0}+w_0+v(\tau)\;\right)dv/d\tau \;=\varepsilon^{n_0-1}[\;q(\varepsilon^{n_0}\tau)\left(w_0+v(\tau)\right)+r(\varepsilon^{n_0}\tau)\;] \tag{20}$$

By virtue of equalities (18) and (19) for $v(\tau)$ we obtain the Cauchy problem:

$$v(\tau_0)=O(\varepsilon^{n_0-1}\,ln\,x\,(1/\varepsilon))\equiv\omega(\varepsilon),\;\;(\tau_0\to\infty,\varepsilon\to 0) \tag{21}$$

$where\quad \tau_0:=\varepsilon^{-n_0\beta}$

Equation (19) is equivalent to the integral equation

$$v(\tau)=\omega(\varepsilon)\;-\;\varepsilon^{n_0-1}\int_\tau^{\tau_0}F(S,v(s),\varepsilon)ds\equiv Tv, \tag{22}$$

where

$$F(\tau,v(\tau),\varepsilon)=\;\left(q(\varepsilon^{n_0}\tau)\left(w_0+v(\tau)\right)+r(\varepsilon^{n_0}\tau)\right)/\left(\tau^{n_0}+w_0+v(\tau)\right) \tag{23}$$

Next, we assume that $w_0>0$. We take a closed ball:

$$C_{\omega(\varepsilon)}=\;\{v\colon\|v(\tau)\|\le 2\omega(\varepsilon)\}.$$

Then we have

$$|F(\tau,v(\tau),\varepsilon)|=\;\left|\left(q(\varepsilon^{n_0}\tau)\left(w_0+v(\tau)\right)+r(\varepsilon^{n_0}\tau)\right)/\left(\tau^{1/n_0}+w_0+v(\tau)\right)\right|\le$$

$$\le\left(M(w_0+2\omega(\varepsilon)+1)/\left(\tau^{1/n_0}+w_0\right)\right)\le\frac{\gamma_1}{\tau^{1/n_0}}, \tag{24}$$

where

$$\gamma_1=2M(w_0+2\omega(\varepsilon)+1),\qquad |q(\varepsilon\tau^{n_0})|,\qquad |r(\varepsilon\tau^{n_0})|\le M.$$

$$|T(\tau)|\;\;\le\omega(\varepsilon)+\varepsilon^{n_0-1}\int_\tau^{\tau_0}|F(S,v(s),\varepsilon)|ds\le\omega(\varepsilon)+\varepsilon^{n_0-1}\int_\tau^{\tau_0}\frac{ds}{S^{1/n_0}}\le$$

$$\le\omega(\varepsilon)+\varepsilon^{n_0-1}\bar{\gamma}_1/\left|\tau_0^{1-1/n_0}\;-\;\tau^{1-1/n_0}\right|$$

$$\le\omega(\varepsilon)+\varepsilon^{n_0-1}\overline{\gamma_1}|\tau_0-\;\tau|^{1-1/n_0}\le$$

$$\le\omega(\varepsilon)+\varepsilon^{n_0-1}\bar{\gamma}_1\cdot\varepsilon^{-(n_0-1)\beta}, \tag{25}$$

where

$\bar{\gamma}_1 = \gamma_1 n_0/(n_0 - 1).$

Now let's choose β – so that it is

$\bar{\gamma}_1 \varepsilon^{-(n_0-1)\beta} = ln(1/\varepsilon), \quad \beta = (ln\frac{1}{\gamma_1} + lnln(1/\varepsilon))/((n_0 - 1)ln(1/\varepsilon).$

Then from (32), we have

$$|Tv| \leq 2\omega(\varepsilon).$$

$$|F_v'(\tau, v(\tau), \varepsilon)|$$

$$= \left|\left(q(\varepsilon^{n_0}\tau)\left(\tau^{1/n_0} + w_0 + v(\tau)\right) - (q(\varepsilon^{n_0}\tau)\left(w_0 + v(\tau)\right)\right.\right.$$

$$\left.\left. + r(\varepsilon^{n_0}\tau)\right) // \left(\tau^{1/n_0} + w_0 + v(\tau)\right)^2\right|$$

$$\leq \left(M/\left(\tau^{1/n_0} + w_0 + v(\tau)\right)\right) +$$

$$+\left(M(w_0 + 2\omega(\varepsilon) + 1)/\left(\tau^{1/n_0} + w_0 + v(\tau)\right)^2\right) \leq$$

$$\leq \left(M/\left(\tau^{1/n_0}\right)\right) + \left(M(w_0 + 2\omega(\varepsilon) + 1)/\left(\tau^{1/n_0} w_0\right)\right)$$

$$\leq \frac{\gamma_2}{\tau^{1/n_0}}, \qquad (26)$$

where

$$\gamma_2 = M + \gamma_1/\, w_0$$

$$|T_1 v_1 - T_1 v_2| \leq \varepsilon^{n_0-1}\left|\int_\tau^{\tau_0} [F(S, v_1(s), \varepsilon) - F(S, v_2(s), \varepsilon)]\, ds\right| \leq$$

$$\leq \varepsilon^{n_0-1}\int_\tau^{\tau_0} |F(S, v_1(s), \varepsilon) - - F(S, v_2(s), \varepsilon)| ds \leq \varepsilon^{n_0-1}\int_\tau^{\tau_0} \frac{|v_1 - v_2|}{F_v(S, v_1(s), \varepsilon)} ds \leq$$

$$\leq \varepsilon^{n_0-1}\|v_1 - v_2\| \int_\tau^{\tau_0} \frac{\gamma_2}{S^{1/n_0}}$$

$$\leq \varepsilon^{n_0-1}\|v_1 - v_2\|\bar{\gamma}_2\left|\tau_0^{1-1/n_0} - \tau^{1-1/n_0}\right| \leq$$

$$\leq \varepsilon^{n_0-1}\bar{\gamma}_2\|v_1 - v_2\||\tau_0 - \tau|^{1-1/n_0} \leq \varepsilon^{n_0-1}\bar{\gamma}_2\bar{\gamma}_1\|v_1 - v_2\|ln(1/\varepsilon).$$

Therefore, we get

$$|T_1 v_1 - T_1 v_2| \leq \varepsilon^{n_0-1} \bar{\gamma}_2 \bar{\gamma}_1 \|v_1 - v_2\| ln(1/\varepsilon), \tag{27}$$

For ε small

$$(\bar{\gamma}_2 \bar{\gamma}_1) \varepsilon^{n_0-1} ln(1/\varepsilon) \leq \frac{1}{2},$$

then the operator T is contractive and we map the ball $C_{\omega(\varepsilon)}$ into itself.

According to the contraction mapping principle, equation (22) has a unique solution on the interval $[0, \tau_0]$, for small ε, where $|v(\tau)| \leq 2\omega(\varepsilon)$.Thus we reach the point $\tau_1^* = \tau_0 - \tau_{N-1} = \tau_0 - \mu$. Similarly, after N steps we reach the point $\tau = 0$.

Thus, the following theorem has been proved.

THEOREM 2. Let in Theorem 1 additionally $w_0 > 0$. Then a solution to problem (1) exists on the segment *[0, 1]* and

$$u(x,\varepsilon) = \begin{cases} u_0(x) + \varepsilon u_1(x) + \varepsilon^2 u_2(x) + \ldots, & \varepsilon^{n_0(1-\beta)} \leq x \leq 1 \\ w_0 + v\left(\dfrac{x}{\varepsilon^{n_0}}\right), & 0 \leq x \leq \varepsilon^{n_0(1-\beta)}. \end{cases}$$

Let, $w_0 < 0$, then $u(x_0, (\varepsilon), \varepsilon) < 0$ for small ε, which follows from an estimate of the form (18). Since $w_0 < 0$ the function $F(\tau, v, \varepsilon)$ is indefinite on the interval $[0, \tau_0]$. Obviously, the solution of problem (1) is not extendable to the point *x=0* for $w_0 < 0$.

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IMPROVED ALGORITHM TO DETECT PATTERNS IN IRGÖÖ-TYPE PROCESSES

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Systematic search of phenomena as consequences of effects was initiated in Kyrgyzstan. Processes of generation of order from chaos (irgöö-type processes by the name of the first process of local diminishing of entropy due to dissolving of energy mentioned in literature) were considered; they are real (including running computers) and random, are defined by some components (states of computer) at each moment. Supra, with the author's participation, appearance of phenomena in systems only with large number of components is said to be the effect of numerosity. The least number of components preserving such phenomenon was said to be the constant related to it. Algorithm to detect simple patterns on a plane was implemented, an improved algorithm is built and applied.

Keywords: algorithm, order, chaos, irgöö, numerosity, effect, phenomenon, differential equation, difference equation

Кыргызстанда эффекттердин катары кубулуштарды ырааттуу издөө башталды.

Баш аламандыктан тартип пайда болгону процесстери (адабиятта биринчи белгилүү болгон энергия тарткатылганынан энтропиянын жергилик азаюу процессинин аталышы боюнча алганда, «иргөө» тибиндеги процесстер) каралат; алардын негизги өзгөчөлүктөрүн төмөнкүчө белгилешет: алар чыныгы (анын ичинде компьютерлердин аракеттери) жана кокустан, ар бир учурда бир нече компоненттер (компьютердин абалдары) менен аныкталат. Мурда, автор катышуусу менен сунуштаган аныктама боюнча, көп элементтерден турган системалар гана үчүн пайда болууучу кубулуштар «көпчө» эффектиси деп аталган. Мындай кубулушка алып келүүчү, ошол кубулуш менен байланышкан, эң кичине сан турактуу болуп саналат. Тегиздикте жөнөкөй саймаларды аныктап таануучу алгоритм ишке ашырылган, өркүндөтүлгөн алгоритм курулат жана колдонулат.

Урунттуу сөздөр: алгоритм, ирет, башаламандык, иргөө, «көпчө», эффект, кубулуш, дифференциалдык теңдеме, айырмалуу теңдеме

В Кыргызстане был начат систематический поиск «явлений» как следствий «эффектов». Рассматриваются процессы возникновения порядка из хаоса (процессы типа «иргөө» по названию первого такого процесса локального уменьшения энтропии вследствие диссипации энергии, известного в литературе), их основные признаки: они - реальные (включая действия компьютеров) и случайны, определяются несколькими компонентами (состояниями компьютера) в каждый момент. Ранее, по определению с участием автора, возникновение явлений только для систем с большим количеством компонент названо эффектом «множественности». Самое малое число, вызывающее такое явление, названо постоянной, связанной с этим явлением. Был реализован алгоритм для

выявления простых узоров на плоскости. Построен и использован усовершенствованный алгоритм.

Ключевые слова: алгоритм, порядок, хаос, иргөө, множественность, эффект, явление, дифференциальное уравнение, разностное уравнение.

1. Introduction

Systematic search of phenomena as consequences of effects was initiated in Kyrgyzstan. Processes of generation of order from chaos (irgöö-type processes by the name of the first such process of local diminishing of entropy due to dissolving of energy mentioned in literature [1]) are considered in the paper. Main features of such processes: they are real (including running computers), random, are defined by some components (states of computer) at each moment.

Considering a computer as a real object and computer presentations as real processes was noted in [2].

Discoveries of new "phenomena" and "effects" used to be sufficient steps in developing science but there were not definitions of these notions before [4] with corresponding definitions and examples, methodic to search new “phenomena” as consequences of “effects”.

Section 2 contains definitions of effect of numerosity, of constant of numerosity and of algorithm to detect phenomena as a constituent of general problem of pattern recognition.

There is the example of some known process in Section 3.

In Section 4 we propose a new algorithm to detect a pattern.

2. Definitions of effect and constants of numerosity and phenomena-detecting algorithm

The law of large numbers can be considered as some phenomena in statistic.

Supra, by our definition, appearance of phenomena in systems only with large number of components was said to be the effect of numerosity.

We found some phenomena due to this effect not related to statistic.

D e f i n i t i o n. Appearance of phenomena in systems only with large number of components is said to be the effect of *numerosity*.

D e f i n i t i o n. If a phenomenon occurs less often for number of components less than N and does more often for number of components greater than N then the number N is said to be the *constant of numerosity* for this phenomenon.

Let any algorithm dealing with a set of numbers given by a certain real experiment or computational experiment with random initial data outputs "yes" or "no" (the algorithm may contain parameters too).

D e f i n i t i o n. If the algorithm outputs "yes" in more than 50% numerous experiments implementing a supposed phenomenon then such phenomenon is existing.

There arises a problem to construct such algorithms for known phenomena.

3. Virtual process "repelling" of irgöö-type

We searched self-ordering of discrete electrical charges in viscous media [3]. Motion of equal, repelling by the Coulomb law electrical charges from a random initial distribution on a topological torus (bounded surface without an edge) formed a final regular grid was modeled by computer.

Motion of N electrical charges can be described by a system of N two-dimensional differential equations. These differential equations were approximated by a system of difference equations. An algorithm was proposed [5] and constructed [6].

4. New algorithm and program

Here we propose new

A l g o r i t h m. Let a finite set of K distinct points $\{z[1].. z[K]\}$ in a bounded metrical (locally Euclidean) space be given. Choose a constant $v>1$.

A) Found the minimums $M[i]:=min\{|z[i]-z[j]|: j\neq i\}, 1\leq i\leq K.$

B) Calculate numbers of neighbors of all points,

$C[i]:=card\{j: M[i]\leq |z[i]-z[j]|\leq M[i]\cdot v\}, i=1..K;$

C) Calculate frequencies, the most frequent one and its frequency

W[q]:= card{i:C[i]=q}, q=1..6;

FW:=max{W[q]: q=1..6}; FI:=argmax{W[q]: q=1..6}; FQ:=FW/K.

D) Output *FI* and *FQ*.

If *FQ> 0.5* then most of numbers *C[i]* are equal and a pattern exists.

For example, in R^2: if *FI=3* then a hexagonal grid exists;

if *FI=4* then a hexagonal grid exists; if *FI=6* then a triangular grid exists.

The following program was written in *pascal*, *v=1.5*.

```
PROGRAM sab_aln; USES CRT, math;
var hxy,vx,vy,dx,dy,dxy,dxy1,hxy1,z,z2,xj,yj,dxy2,dxyd,
d_xy2,mn,rel_xy,scous,v: real; i,j,nxy,it,nt,np,ihand,n_time,ik: longint;
ncount: array[1..500] of integer; w: array[0..100] of integer;
var f,n,iw,jcase: integer; x,y:array[1..500] of real;
function dxy_2(ii,jj:longint): real; begin
xj:=x[jj]; if xj>x[ii]+z2 then xj:=xj-z; if xj<x[ii]-z2 then xj:=xj+z;
yj:=y[jj]; if yj>y[ii]+z2 then yj:=yj-z; if yj<y[ii]-z2 then yj:=yj+z;
dxy_2:=sqr(x[ii]-xj)+sqr(y[ii]-yj); end;
begin {main} randomize;
writeln('  Tagaeva, Dec. 2022. Repelling charges on torus, improved');
for jcase:=1 to 5 do begin write(' Give number of charges and wait a little: ');
readln(nxy); v:=1.5; z:=700.; z2:=z/2.0; np:=10; hxy:=1.0; hxy1:=hxy; nt:=1000;
for ik:=1 to nxy do begin  x[ik]:=z*random; y[ik]:=z*random; end;
for it:=0 to nt do begin {it} if it>np then hxy:=2.0*hxy1;
if it>2*np then hxy:=4.0*hxy1;
for i:=1 to nxy do begin {i=ix} vx:=0.; vy:=0.; for j:=1 to nxy do
begin if j<>i then begin dxy2:=dxy_2(i,j)+1.;
dxy1:=z/(dxy2*sqrt(dxy2)); if dxy1<sqr(z)/nxy then begin
dx:=(x[i]-xj)*dxy1; dy:=(y[i]-yj)*dxy1; vx:=vx+dx; vy:=vy+dy; end; end; end;
x[i]:=x[i]+vx*hxy; if x[i]>z then x[i]:=x[i]-z; if x[i]<0. then x[i]:=x[i]+z;
y[i]:=y[i]+vy*hxy; if y[i]>z then y[i]:=y[i]-z; if y[i]<0. then y[i]:=y[i]+z;
                                        end {i=ix}; end {it};
```

```
for iw:=0 to 100 do w[iw]:=0; for i:=1 to nxy do begin mn:=100000.0;
for j:=1 to nxy do begin if i<>j then mn:=Min(mn, dxy_2(i,j)) end;
ncount[i]:=0;
for j:=1 to nxy do begin if (i<>j) and (dxy_2(i,j)<mn*v) then
ncount[i]:=ncount[i]+1 end; end;
for j:=1 to nxy do w[ncount[j]]:=w[ncount[j]]+1;
for iw:=2 to 7 do begin write(' ',iw:1,' ',w[iw]:2,';') end; writeln; end; readln; END.
```

This program was run many times to calculate the most frequent number W of neighbors for each N being a square of integer.

N: 81 100 121 144 169 196 225

W: 5,6 4,5 5,6 4 6 4 6

Hence, the strict constant of numerosity is ~ 144 (beginning of evident alternation).

Also, when the number of charges is a square of even number then the grid is square in most of experiments; when it is a square of odd number then the grid is triangular in most of experiments.

Conclusion

We hope that proposed definitions would yield new phenomena in reality and in computational experiments and constants of numerosity would be found for other real and virtual processes. The general problem: what kinds of patterns can be detected by any algorithms?

The work was carried out within the framework of the project of the Institute of Mathematics of the National Academy of Sciences of the Kyrgyz Republic.

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FUNCTIONAL RELATIONS AND MATHEMATICAL MODELS OF VARIABLE OBJECTS

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Many objects to be presented on computers for scientific and educational purposes are variable. For convenient algorithmic presentation, we propose to use functional relations. For independent interactive computer presentation of natural languages, B. Bayachorova and S. Karabaeva proposed to present transforming verbs. Such verbs are more complex than “sequences of shifts”. In the paper mathematical and computer models of some notions are constructed by functional relations between points of virtual objects.

Keywords: variable object, language, computer presentation, mathematical model, independent presentation

Көп компьютерде илимий жана окутуу максаттары үчүн көрсөтүлүүчү объект өзгөрүүчү болуп эсептелет. Ыңгайлуулук үчүн алгоритмдик чагылдырууда функционалдык өз ара байланыштарды колдонууну сунуштайбыз. Табигый тилдерди компьютерде көз карандысыз интерактивдик чагылдыруу үчүн Б.Баячорова жана С.Карабаева өзгөртүүчү этиштерди чагылдырууну сунуш кылган. Мындай этиштер, “жылдыруулардын удаалыштыгына” караганда, татаалыраак. Бул макалада кээ бир түшүнүктөрдүн математикалык жана компьютердик моделдери элестетилген (виртуалдуу) объекттердин чекиттеринин арасындагы функционалдык өз ара байланыштар аркылуу курулду.

Урунттуу сөздөр: өзгөрүүчү объект, тил, компьютердик чагылдыруу, математикалык модель, көз карандысыз чагылдыруу

Многие объекты, которые нужно представить на компьютере для научных и учебных целей, являются переменными. Для удобного алгоритмического представления мы предлагаем использовать функциональные соотношения. Для независимого интерактивного компьютерного представления естественных языков Б.Баячорова и С.Карабаева предложили представить преобразующие глаголы. Такие глаголы - более сложные, чем «последовательность сдвигов». В данной статье построены математические

и компьютерные модели некоторых понятий при помощи функциональных соотношений между точками виртуальных объектов.

Ключевые слова: переменный объект, язык, компьютерное представление, математическая модель, независимое представление

1. Introduction

One of main directions of modern informatics is construction of computer presentations of various objects and processes for scientific and educational purposes. Many of such objects can be presented by consequences of “shifts” or “Drag-and-

Drop” operations.

For independent interactive computer presentation of natural languages, presenting transforming verbs was proposed in [11]. Such verbs are more complex than “sequences of shifts”. In [13] we noted that mathematical and computer models of such verbs can be constructed by functional relations [12] between points of virtual objects. In this paper we consider this item in general.

1. Review of using objects for teaching languages

At all times, travellers learned languages from inhabitants which did not know the traveller’s native language. This method (demonstration of objects and actions with comments) was described in details in “Gulliver's travels” by J.Swift, 1726. In 1878 M.Berlitz proposed a method with is considered as “the first-known immersive teaching method”. Such method was improved as “Total physical response” [1].

Winograd T. [2] proposed giving commands to a robot with such words as "table", "box", "block", "pyramid", "ball", "grasp", "moveto", "ungrasp".

Using these ideas, since [3] general methods for interactive computer presentations of natural languages are being developed [4-10]. If a computer presentation does not depend on the user’s knowledge and skills on similar objects then it is called *independent*. Such presentations are more effective because the user can learn a language inductively, and they begin thinking in it, without translation in mind.

2. Definitions for independent presentation of objects

The following definitions were proposed in [4-12].

Definition 1. If low energetic outer influences can cause sufficiently various reactions and changing of the inner state of the object (by means of inner energy of the object or of outer energy entering into object besides of such influences) at any time then such (permanently unstable) object is an *affectable object*, or a *subject*, and such outer influences are *commands*. A system of commands such that any subject can achieve desired efficiently various consequences from other one is a *language*.

The following definition is proposed by us.

Definition 2. A marked set (2D-set on display) is a set (S) with some marked points on it $(x_1, x_2, x_3 \ldots \in S)$. A transformable object is a (varying) set or union of marked sets, marked points of them are connected with *functional relations:* "$\zeta(x,y)$".

R e m a r k. Marked sets were proposed in [14] and developed in [15].

Definition 3. Mathematical models consist - of fixed (F_i) and movable (M_j) sets; an extended algorithm: temporal sequence of conditions of types

$(M_j \subset F_i)$, $(M_j \cap F_i = \varnothing)$, $(M_j \cap F_i \neq \varnothing)$; $\zeta(x_k(M_j), x_q(M_j))$.

R e m a r k. Mathematical models also include *transforming* objects.

Definition 4. Transformation of objects can be presented by a controlled differential equation. Firstly, we consider motion of a flat object without inertia and not-self-moving objects. Let $S \in R^2$ be initial position of the object; $S(t)$ be the position and shape of the object at time t. We have the equation $y'(t,z)=F(t, S(t),u(t))$, $0 \leq t < \infty$ with initial condition $y(0)= z \in S$, where $u(t)$ is a control function (given by the user), the function $F(s,u)$ (to be implemented by the programmer) is bounded, $y(t,z)$: $[0, \infty)\times S \to R^2$ is the trajectory of the point $z \in S$. Let S and $S(t)$ be marked, defined by points $z_1, z_2,\ldots,z_k \in S$ and their images correspondingly. Then $F=F(t,y_1, y_2,\ldots,y_k,u(t))$. Values $y_1, y_2,\ldots,y_k$ are to be connected with functional relations. Computer interactive presentations are built on the base of mathematical models.

3. Mathematical models for transformable objects

3.1. $k=2$, the functional relation is $dist(y_1,y_2)=const$. It can be used for verbs ROTATE, TURN; PULL.

3.2. $k=3$, the functional relations are $y_1=const$, $y_2=const$, $dist(y_2,y_3)=const$. It can be used for the verb BEND.

3.3. $k>3$, the functional relations are

$$dist(y_1,y_2)= dist(y_2,y_3)=\ldots= dist(y_{k-1},y_k)= const$$

It can be used for the noun LENGTH, verb BEND.

3.4. $k\geq2$. Firstly, two S_1 and S_2. $y_1\in S_1$ and $y_2 \in S_2$. The relation is $y_1=y_2$ for any $t>t_1$. (Or more than two distinct objects).

3.5. $k=2$. The functional relations are $y_1=const$, $dist(y_1,y_2)$ increases. The verb STRETCH.

4. Conclusion

This paper enlarges the scope of objects which can be presented interactively in the frames of general project of developing mathematical models of various notions for independent presentation of natural languages. We hope that such software would be interesting and useful for people to study various objects interactively.

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ON A CANONICAL PROJECTOR OF A VECTOR IN AN IDEMPOTENT SPACE BY A SEMIMODULE OF IDEMPOTENT PROBABILITY MEASURES

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In the paper we show general form of a canonical projector of a vector in an idempotent space by a semimodule of idempotent probability measures.

Keywords: semimodule, canonical projector.

Бул макалада биз идемпотенттүү мейкиндик векторунун модулунун идемпотенттик ыктымалдык өлчөмдөрүнүн канондук проекторунун жалпы көрүнүшүн көрсөтөбүз.

Уурунттуу сөздөр: жарым модуль, канондук проектор.

В данной работе показан общий вид канонического проектора вектора идемпотеного пространства по полумодулю идемпотентных вероятностных мер.

Ключевые слова: полумодуль, канонический проектор.

Subsemimodules and convex subsets of semimodules over semirings with an idempotent addition studied by Guy Cohen *et. all* 2003 [1]. Furthermore appeared Projector Formula and proved Universal Separation Theorem for an idempotent complete subsemimodule (see [2] for details). In this paper, we give a Projector Formula and an Universal Separation Theorem for the idempotent semimodule $I(X)$.Let $\mathbb{R}_{\max} = \mathbb{R} \cup \{-\infty\}$ equip with two operations $\vee$ and $\odot$, which are defined as $x \vee y = \max\{x, y\}$, $x \odot y = x + y$, x, $y \in \mathbb{R}_{\max}$. Then $\mathbf{0} := -\infty$ is the zero in $\mathbb{R}_{\max}$ according to $\oplus$, and $\mathbf{1} := 0$ is the unit in $\mathbb{R}_{\max}$ according to $\odot$. It is well-known [3] that $(\mathbb{R}_{\max}, \oplus, \odot, \mathbf{0}, \mathbf{1})$ is idempotent semifield (since $x \vee x = x$ for all $x \in \mathbb{R}_{\max}$).

By ordered set, we will mean throughout the paper a set equipped with a partial order. Let a partially order define the following way:

$$a \le b \quad \Leftrightarrow a \vee b = b.$$

For any subset Y of an ordered set $(S,\leq)$ by $\vee Y$ (resp. $\wedge Y$) we denote the least upper bound (resp. greatest lower bound) of Y, when it exists [2]. When $\vee Y$ (resp. $\wedge Y$) belongs to Y, $\vee Y$ ($\wedge Y$) is said to be the *top* (resp. *bottom*) element of Y, and written $\top Y$ (resp. $\perp Y$) instead of $\vee Y$ (resp. $\wedge Y$).

Definition 1.[4] Let V be an idempotent semigroup, K be an idempotent semiring, and let a multiplication operation $k,x \mapsto k \odot x$ be defined so that the following equalities hold

$$(k_1 \oplus k_2) \odot x = (k_1 \odot x) \oplus (k_2 \odot x),$$

$$k \odot (x \oplus y) = (k \odot x) \oplus (k \odot y),$$

$$1 \odot x = x$$

for all $x, y \in V$, $k, k_1, k_2 \in K$. Then the semigroup V is called a (left) idempotent semimodule over the semiring K (or simply a semimodule).

Definition 2.[4] A semimodule V over a semifield K is called an idempotent space.

This notions is analogous to the notion of linear (vector) space over a field.

Idempotent mathematics is based on replacing the usual arithmetic operations with a new set of basic operations, i. e., on replacing numerical fields by idempotent semirings and semifields. Typical example is the so-called max-plus algebra $\mathbb{R}_{\max}$. Many authors (S. C. Kleene, S. N. N. Pandit, N. N. Vorobjev, B. A. Carre, R. A. Cuninghame-Green, K. Zimmermann, U. Zimmermann, M. Gondran, F. L. Bacelli, G. Cohen, S. Gaubert, G. J. Olsder, J.-P. Quadrat, V. N. Kolokoltsov, A. A. Borubaev, A. A. Zaitov and others) used idempotent semirings and matrices over these semirings for solving some applied problems in computer science and discrete mathematics, starting from the classical paper by S. C. Kleene [5].The modern idempotent analysis (or idempotent calculus, or idempotent mathematics) was founded by V. P. Maslov and his collaborators.

Idempotent mathematics can be treated as the result of a dequantization of the traditional mathematics over numerical fields as the Planck constant h tends to zero taking imaginary values. In other words, idempotent mathematics is an asymptotic version of the traditional mathematics over the fields of real and complex numbers.

The basic paradigm is expressed in terms of an idempotent correspondence principle. This principle is closely related to the well-known correspondence principle of N. Bohr in quantum theory. Actually, there exists a heuristic correspondence between important, interesting, and useful constructions and results of the traditional mathematics over fields and analogous constructions and results over idempotent semirings and semifields (i. e., semirings and semifields with idempotent addition) [5].

Let X be a set consisting of n different points, and equipped with the discrete topology. Consider the following sets:

$$P(X)=\left\{\sum_{i=1}^{n}\mu_i\delta_{x_i} : \mu_i\geq 0, i=1,2,\ldots,n, \sum_{i=1}^{n}\mu_i=1\right\},$$

$$I(X)=\left\{\bigoplus_{i=1}^{n}\mu_i\odot\delta_{x_i} : \mu_i\geq -\infty, i=1,2,\ldots,n, \bigoplus_{i=1}^{n}\mu_i=0\right\}.$$

It is well-known that $P(X)$ represents the set of all probability measures on X, and $I(X)$ represents the set of all idempotent probability measures on X [3]. Moreover, $P(X)\subset\mathbb{R}^n$ and $I(X)\subset\mathbb{R}^n_{\max}$.

In figure 1 we show the difference of this sets.

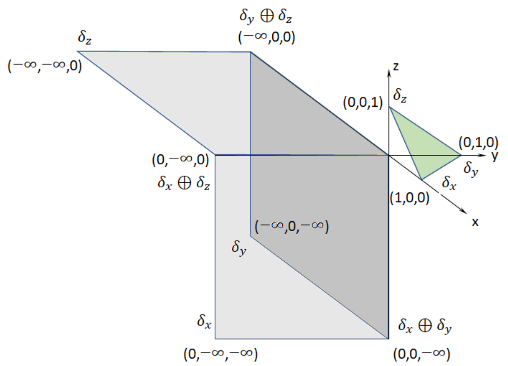

Figure 1. The images $I(X)$ (grey part) and $P(X)$ (green part) when X consists of three points.

Let Y be a complete semimodule, $\mathcal{K}$ a complete semiring. For x, $y\in Y$ and $\lambda\in\mathcal{K}$ put

$$x\setminus y\overset{def}{=}(L_x^Y)^{\#}(y)=\top\{\lambda\in\mathcal{K}\,|\,x\odot\lambda\leq y\},$$

$$x/\lambda\overset{def}{=}(R_\lambda^Y)^{\#}(x)=\top\{z\in Y\,|\,z\odot\lambda\leq x\}.$$

Nonlinear projector. Let V be a complete subsemimodule of a complete semimodule Y over a complete idempotent semiring $\mathcal{K}$, i. e., a subset of Y that is stable by arbitrary sups and by the action of scalars. A map $P_V:Y\to Y$ is called [2] a canonical projector on V which defines

$$P_V(y) = \top\{v \in V \mid v \le y\}, \qquad y \in Y.$$

W is called [2] a *generating family* of a complete subsemimodule V if any element $v \in V$ can be written as $v = \vee\{w \odot \lambda_w \mid w \in W\}$, for some $\lambda_w \in \mathcal{K}$.

Example 1. $\mathbb{R}^n_{\max}$ is a semimodule over $\mathbb{R}_{\max}$, $I(X)$ is a subsemimodule of $\mathbb{R}^n_{\max}$, and the family

$$W: \begin{cases} \delta_{x_1} = (\mathbf{1}, \mathbf{0}, \ldots, \mathbf{0}), \\ \delta_{x_2} = (\mathbf{0}, \mathbf{1}, \ldots, \mathbf{0}), \\ \quad \ldots \quad \ldots \quad \ldots \\ \delta_{x_n} = (\mathbf{0}, \ldots, \mathbf{0}, \mathbf{1}) \end{cases}$$

is generating family of $I(X)$ for $\mathbf{0} \le \lambda \le \mathbf{1}$.

It is obvious that for $I(X)$ and any $y \in \mathbb{R}^n_{\max}$ the canonical projector $P_{I(X)}(y)$ has the following three cases

$a)$ if $y = (y_i)_{i\in[n]} \ge 0$ then $P_{I(X)}(y) = \top\{\mu \in I(X) \mid \mu \le y\} = \underset{i\in[n]}{\vee} \mathbf{0} \odot \delta_{x_i}$;

$b)$ if y and 0 are not comparable then $P_{I(X)}(y) = \top\{\mu \in I(X) \mid \mu \le y\} \in I(X)$;

$c)$ if $y = (y_i)_{i\in[n]} < 0$ then y does not have a projector by $I(X)$, i. e.

$$P_{I(X)}(y) = \top\{\mu \in I(X) \mid \mu \le y\} = \top\varnothing.$$

Theorem 1. (Projector Formula) For the set $I(X)$ with generating family W, the following equality holds $P_{I(X)}(y) = \underset{w\in W}{\vee} (w \setminus y) \odot w.$

Here $y = (y_i)_{i\in[n]} \in \mathbb{R}^n_{\max}$, and there exists at least an index $i_0 \in [n]$ such that $y_{i_0} \ge 0$.

Proof. Since W is a generating family of the semi module $I(X)$, for every $y \in \mathbb{R}^n_{\max}$ its projector by $I(X)$ can be rewritten as $P_{I(X)}(y) = \underset{i\in[n]}{\vee} \lambda_i \odot \delta_{x_i}$, here $\lambda_i \in \mathbb{R}_{\max}$.

Suppose $y = (y_i)_{i\in[n]} \ge 0$. Then for every $w \in W$ we have

$$w \setminus y \equiv \delta_{x_i} \setminus y = (\mathbf{0}, \ldots, \mathbf{1}, \ldots, \mathbf{0}) \setminus (y_1, \ldots, y_i, \ldots, y_n) = \mathbf{1}.$$

In this case $P_{I(X)}(y) = \underset{i\in[n]}{\vee} \mathbf{1} \odot \delta_{x_i} = \underset{w\in W}{\vee} (w \setminus y) \odot w$.

Let now y and 0 are not comparable. Then

$$w \setminus y \equiv \delta_{x_i} \setminus y = (\mathbf{0},\ldots,\mathbf{1},\ldots,\mathbf{0}) \setminus (y_1,\ldots,y_i,\ldots,y_n) = \begin{cases} \mathbf{1}, & if \quad y_i \geq \mathbf{1}, \\ y_i, & if \quad y_i < \mathbf{1}. \end{cases}$$

Therefore, by definition we have $P_{I(X)}(y) = \underset{w\in W}{\vee} (w \setminus y) \odot w$. Theorem 1 is proved.

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PATHS INTERSECTION AND EXISTENCE OF MULTIDIMENSIONAL LOCALLY HETEROGENEOUS KINEMATICAL SPACES

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There are found conditions preserving existence of a point in a multi-dimensional kinematical space with a neighbor to be non-homeomorphic to a ball; that is the space is heterogeneous.

Keywords: kinematical space, path, heterogeneity, neighbor, condition

Кинематикалык көп өлчөмдүү мейкиндикте шарга гомеоморфтук эмес айланасы болгон чекит бар болуу (б.а. мейкиндик бир тектүү эмес) экендигин камсыз кылуучу шарттар табылган.

Урунттуу сөздөр: кинематикалык мейкиндик, жол, бир тектүү эместик, айлана, шарт.

Найдены условия, при которых в многомерном кинематическом пространстве существует точка с окрестностью, негомеоморфной шару, то есть пространство оказывается неоднородным.

Ключевые слова: кинематическое пространство, путь, неоднородность, окрестность, условие.

1. Introduction

Definition of a kinematical space M (the metric ρ_K is the minimal time of passing between points) was introduced in [1]. There exist paths in such spaces, length of a path $[a\text{-}b]$ between distinct points a and b is not less than $\rho_K(a,b)$.

D e f i n i t i o n 1 [2]. If two points of a topological space have homeomorphic neighborhoods then they are said to be locally homogeneous.

We proposed

D e f i n i t i o n 2. If a topological space has at least two non-homogeneous points then it is said to be locally non-homogeneous.

Remark. Some conditions on non-constructive existence of non-homogeneous spaces are given in [3], [4], [5].

2. Definitions for motion of points with bounded velocity

As a codifying the ancient idea of controlled motion with bounded velocity, the notion of kinematical space was introduced [1].

D e f i n i t i o n 3. A computer program is said to be a **presentation** of a computer kinematical space if:

P1) there is an (infinite) metrical space X of points and a set X_1 of display-presentable points being sufficiently dense in X;

P2) the user can pass from any point x_1 in X_1 to any other point x_2 by a sequence of adjacent points in X_1 by their will;

P3) the minimal time to reach x_2 from x_1 is (approximately) equal of the minimal time to reach x_2 from x_1.

The space X is said to be a **kinematic space**; the space X_1 is said to be a **computer kinematic space**; this minimal time is said to be the **kinematical distance** ρ_X between x_1 and x_2; a sequence of adjacent points is said to be a **route**. Passing to a limit as X_1 tends to X we obtain the following.

There is a set K of **routes**; each route M, in turn, consists of the positive real number T_M (**time** of route) and the function $m_M : [0, T_M] \to X$ (**trajectory** of route);

(K1) For $x_1 \neq x_2 \in X$ there exists such $M \in K$ that $m_M(0) = x_1$ and $m_M(T_M) = x_2$, and the set of values of such T_M is bounded with a positive number below;

(K2) If $M = \{T_M, m_M(t)\} \in K$ then the pair $\{T_M, m_M(T_M - t)\}$ is also a route of K (the reverse motion with same speed is possible); (cf. P3).

(K3) If $M = \{T_M, m_M(t)\} \in K$ and $T^* \in (0, T_M)$ then the pair: T^* and function $m^*(t) = m_M(t)$ $(0 \leq t \leq T^*)$ is also a route of K (one can stop at any desired moment);

(K4) concatenation of routes for three distinct points x_1, x_2, x_3.

If there exists a kinematic consistent with the given metric then the metric space is said to be **kinematizable**.

Denote *[a-b]* as any path (trace of a rout) in a kinematic space X connecting points a and b.

Denote $S_K(a, r):=\{y \in X: \rho_K(a, y)< r\}$.

3. One-dimensional kinematical spaces

T h e o r e m 1. If there are such point $z \in X$ and $r>0$ that $S_K(a, r)$ is isometric to $(-r, r)$ and four distinct points $a, b, c, d \in X$ such that any paths *[a-b], [a-c], [a-d]* have the only common point

then the kinematical space X is not locally homogeneous.

P r o o f. Let $r < min\,\{\rho_K(a, b), \rho_K(a, c), \rho_K(a, d)\}$.

If $S_K(a, r)\cap [a-b]$ is not isometric to $(-r, 0]$ then the kinematical space X is not locally homogeneous, as well as $S_K(a, r)\cap [a-c]$ and $S_K(a, r)\cap [a-d]$.

If all these three sets are isometric to $(-r, 0]$ then $S_K(a, r)$ *is* isometric to the union of three copies of $(-r, 0]$ glued at the point 0 and the point a is not locally homogeneous to the point z. Q.E.D.

4. Main theorems on multi-dimensional kinematical spaces

Denote $S_K(a, r):=\{y \in M: \rho_K(a, y)< r\}$.

T h e o r e m 2. In a kinematic space M, if each point x has a neighbor $S(x, r)$ being isometric to an open ball of radius r in R^n , $n\geq 2$, then for any path $[a-b]_1$ and compact set C_2, $[a-b]_1\cap C_2 = \varnothing$ there exists (another) path $[a-b]_2$ and its finite covering

$$S^*:= \cup_{k=1}{}^{m} S_K(x_k,r_k)$$

such that $S(x_i,r_i) \cap S (x_{i+1}, r_{i+1})\neq \varnothing$, $i =1..m-1$; $S^*\cap C_2 =\varnothing$ and for the boundary $Bd(S^*) \cap [a-b]_2 =\varnothing$.

P r o o f. By the condition, for each $x \in [a-b]_1$ there exists a neighbor $S(x, r)$. Diminish each r to ensure $r < \rho_K(x,C_2)$. Due to compactness, there exists a finite cover of the set $[a-b]_1$ by these open balls $S(x,r)$. Number them along the path *[a-*

$b]_1$. Further, we change segments $[x_i,x_{i+1}]$ of the path to ones with lengths close to $\rho_K(x_i,x_{i+1}), i = 1,\dots,m-1$.

T h e o r e m 3. If 1) there are four distinct points $a, b, c, d \in X$ such that there exists a path $[a\text{-}c]$ not containing points b and d and every path $[a\text{-}c]$ and every path $[b\text{-}d]$ have a common point; 2) At least one point has a neighbor isometric to an open ball in R^n then the space X is locally non-homogeneous.

P r o o f (by contradiction). Let all points in X have neighbors isometric to open balls in R^n, let a point $e \in [a\text{-}c] \cup [b\text{-}d]$. Let $\{x_1,x_2,\dots,x_n\}$ be a sequence built by Theorem 1 with $C_2:=\{\{b\}, \{d\}\}$. As b and d are not in $Bd(S^*)$, let z_0 be the "first" point of a path $[b\text{-}d]$ in $Bd(S^*)$ and z_1 be the "last" one, i and j are such numbers that $z_0 \in S(x_i, r_i)$ and $z_1 \in S(x_j, r_j)$. Without loss of generality, $i \le j$.

Choose points $y_k \in Bd(S(x_k, r_k)) \cup Bd(S(x_{k+1}, r_{k+1}))$, $k=i,\dots,j$ and draw a path along boundaries:

$$[z_0\text{-}z_1] \equiv [z_0\text{-}y_i\text{-}y_{i+1}\text{-}\dots\text{-}y_{j-1}\text{-}z_1].$$

Hence, there exists the path $[b\text{-}z_0\text{-}z_1\text{-}d]$ having empty intersection with the path $[a\text{-}c]$ and, hence, which contradicts the condition 1) of Theorem. Q.E.D.

5. Corollary

The following theorem can be considered as application of

Dirichlet's box principle: if p items are put into q containers, with $p > q$, then at least one container must contain more than one item.

T h e o r e m 4. If 1) there are distinct points $a, b, c_1, c_2, \dots, c_q, d \in X$ such that there exist paths $[a\text{-}c_1], [a\text{-}c_2], \dots, [a\text{-}c_q]$, each couple of them has the only common point a; 2) one of paths $[a\text{-}c_j]$ and every path $[b\text{-}d]$ have a common point; 3) At least one point has a neighbor isometric to an open ball in R^n
then the space X is locally non-homogeneous.

P r o o f. Suppose that the paths $[a\text{-}c_j]$ have a common point $[b\text{-}d]$. Applying Theorem 3 we obtain locally non-homogeneity.

6. Conclusion

For kinematical spaces being subspaces of R^n, these theorems provide existence of points z such that for all its neighbors V the set difference $V\backslash\{z\}$ is not connected. Probably, points with neighbors being holomorphic to "half-balls" for $n=2$

$$\{(z_1,z_2,\ldots,z_n)\in R^n : z_1^2+z_2^2+\ldots+z_n^2\leq r^2,\ z_1\geq 0\}$$

by means of paths "which cannot be crossed".

What other types of neighbors can be detected by properties of paths?

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MORPHISMS IN CATEGORY OF CONTROLLED PROCESSES IN COMPUTATIONAL MATHEMATICS

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Supra, the notion of category of equations was introduced by the second author with assistance of the notion “predicate” on the base of the principle of preservation of solution while transformations; elements of the category of equations and its subcategories were constructed on the base of well-known categories. As a specification of the second law of thermodynamic for isolated systems, supra the first author introduced new definitions, proposed general hypotheses and derived estimations from below on increasing of entropy on depending on time in permanently unstable (affectable) systems. On this base, the authors proposed the category of controlled processes
in computational mathematics. Definition and examples of morphisms in this category are proposed in this paper.

Keywords: category, morphism, entropy, control, differential equation, affectable system, motion.

Мурда экинчи автор, өзгөртүүлөрдө чыгарылышты сактоо принцибинин негизинде “предикат” түшүнүгүнүн жардамы менен теңдемелердин категориясынын түшүнүгүн киргизген; теңдемелердин категориясынын жана анын категориячаларынын элементтери белгилүү категориялардын негизинде курулган. Четтетилген системада термодинамиканын экинчи законун тактоо катары, биринчи автор жаңы аныктамаларды киргизген, жалпы гипотезаларды сунуштап, таасир этилүүчү системада материалдык чекитти сүрүлүүсүз жана сүрүлүүнүн негизинде кандайдыр бир аралыкка жылдырууда убакыттан көз каранды болгон энтропиянын өсүүсүнүн төмөнкү баасын алган. Бул макалада бул аныктамалар айкалыштырылган.

Урунттуу сөздөр: категория, морфизм, энтропия, башкаруу, дифференциалдык теңдеме, таасир этилүүчү система, кыймылдоо.

Ранее вторым автором было введено понятие категории уравнений с помощью понятия “предикат” на основе принципа сохранения решения при преобразованиях;

построены элементы категории уравнений и ее подкатегорий на основе известных категорий.

Как уточнение второго закона термодинамики для изолированных систем, первый автор ввел новые определения, предложил общие гипотезы и получил оценки снизу для возрастания энтропии при передвижении материальной точки без трения и с трением на определенное расстояние в зависимости от времени в перманентно неустойчивых системах. В данной статье эти определения объединены.

Ключевые слова: категория, морфизм, энтропия, управление, дифференциальное уравнение, перманентно неустойчивая система, движение.

1. Introduction

Investigations in the category of topological spaces in Kyrgyzstan were initiated [1]. The notion of the category of equations was introduced [2] with assistance of the notion “predicate” on the base of the principle of preservation of solution while transformations; elements of the category of equations and its subcategories were constructed on the base of well-known categories.

As an improving of the second law of thermodynamic for isolated systems, on the base of [3], [4], [5], we put a general hypothesis on estimations from below on increment of entropy during motions in [6].

By using new definitions in [7] If low energetic outer influences can cause sufficiently various reactions and changings of the inner state of the system then it is said to be an “almost isolated”, “permanently unstable”, “affectable” system (A-system); such outer influences are said to be commands (these reactions and changings are implemented by means of inner free energy of the object) we found some estimations from below on increment of entropy while motion of a material point both without friction and with friction over definite distance on depending on time in such systems.

Section 2 contains survey of known categories including the category of energy-entropy-optimization processes.

There are examples of some morphisms in this category in Section 3.

Remark. Morphisms are denoted as *Mor* or *Hom* in literature.

2. Survey of known categories

2.1) The basic category *Set* of sets. *Ob(Set)* are sets; *Mor(Set)* are functions.

2.2) The category *Func* of functions (synonyms in various branches of mathematics: maps, operators, transformations). It is mentioned in publications but we did not find its formal description. *Ob(Func) = Mor(Set), Mor(Func)* are transformations of functions.

2.3) The category *Top* of topological spaces. *Ob(Top)* are topological spaces, *Mor(Top)* are continuous functions.

We proposed

2.4) Category of equations *Equa*. *Ob(Equa)* contains tuples

{Non-empty sets X, Y, predicate $P(x)$ on X, transformation $B: X \to Y$}.

If $(\exists x \in X)(P(x) \wedge (y=B(x))$ then $y \in Y$ is said to be a solution of the equation $\{X, Y, P, B\}$. Particularly, if B is the identity operator I, then we obtain the equation *"P(x)"* only. *Mor(Equa)* are such transformations of tuples $\{X, Y, P, B\}$ that solutions (or their absence) preserve.

We also defined some subcategories of *Equa:*

2.5) The category *Equa-Func* of equations for functions.

2.6) The category *Equa-Par* of equations with parameters.

2.7) The category *Equa-Func-Par* of equations for functions with parameters (including control).

2.8) The category *Equa-Func-Ent* of energy-entropy-optimization controlled processes.

Denote the physical dimension as *dim; dim(time)*= τ; *dim(length)*=λ; *dim(mass)*= μ; $dim\,(absolute\ temperature\,\Theta) = \vartheta$. Then *dim(energy)*= $\mu\lambda^2/\tau^2$.

In processes considered below increment of entropy is defined as $\Delta H = \sum \Delta Q/\Theta$ where ΔQ is energy that was converted to heat irreversibly; $dim(\Delta H) = \mu\lambda^2/\tau^2/\vartheta$.

The second law of thermodynamic for isolated systems states increment of entropy $\Delta H > 0$ but does not give quantitative estimations. One estimation was proven in [4]: the minimal energy (increment of entropy) to treat one bit of information is the Shannon-von Neumann-Landauer boundary:

$\Delta H_{bit} = k_B \ln 2$ where k_B is the Boltzmann constant.

A hint to such estimation in general case was in [5]: “In any mechanical system the energy that must be expended to work against friction is equal to the product of the frictional force and the distance through which the system travels. Hence the faster a swimmer travels between two points, the more energy he or she will expend, although the distance traveled is the same whether the swimmer is fast or slow.”

Basing on the notion of economical (cruising) speed, taking into account ide-as of Ecological Rallies for cars we specified the second law of thermodynamic for A-systems. We proved some estimations in mathematical models describing such concrete systems [8]-[9]-[10].

Let there is a (closed) system. Let it is in any stationary state A now and there it can pass to any other stationary state B.

Hypothesis [6]. There exists such time T_0 (the adiabatic time of the system), depending only on the initial state of the system, that ΔH is not less than any positive value for any transition from the state A to the state B during $T<T_0$. Moreover, there also exists such positive constant C_0 that $\Delta H \geq C_0 /T^{2}$; $dim(C_0)= \mu\lambda^2/\vartheta$.

By the principle of determinism, there is only scenario of the future for any iso-lated system, that is, there cannot be different possibilities of transitions. Hence, the system is to be A-system (with unbounded volume of inner free energy): different possible actions, transferring it from the state A to the state B by means of irreversibly converting free energy to heat which are controlled by any outer impetus of sufficie-ntly small energy.

Ob(Equa-Func-Ent) are A-systems with the task (*): to transfer the state A to the state B in time T with minimal increment of entropy ΔH.

Mor(Equa-Func-Ent) are transformations of A-systems preserving the minimal increment of entropy.

3. Examples of morphisms

<u>Example 3.1</u>. Let B be over A in gravitation field with constant q, $dim\ q =\lambda/\tau^{2}$. There is the load M of mass m based on many compressed short almost

ideal springs at the point *A*, the massive brake with temperature Θ along the segment *AB* and the catcher at the point *B; (*).*

It is proven in [11] that the adiabatic time is $T_0 := \sqrt{(2h/q)}$ and

sup $\Delta H = mh^2/T^2\,(1- T^2/\,T_0{}^2)^2/2/\Theta.$

Example 3.2. Let *A* be over *B* at the height *h* in gravitation field and *B* be on an

absolutely elastic plane. There is the load of mass *m* at the point *A*, the massive brake with the absolute temperature Θ along the segment *AB* and the catcher at the point *A*. We can apply unbounded force to the load. It is necessary to return the load to the point *A* by repelling at the point *B* during the time *T*.

It is proven in [11] that the adiabatic time is $T_0 := 2\sqrt{(2h/q)}$ and

sup $\Delta H = 2mh^2/T^2\,(1- T^2/\,T_0{}^2)^2/\Theta.$

Theorem 1. Changing $(m, h, q) \to (m/\eta^2, \eta h, \eta q)$, $\eta>0$ yields a morphism.

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TERMS OF ARITHMETIC PROGRESSION AS IDENTIFIERS IN THE DECOMPOSITION METHOD

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It is revealed that the main mathematical apparatus for constructing high-order M-matrices is an arithmetic progression. By virtue of the properties of the constants of squares as a member of an arithmetic progression, we obtain the possibility of constructing matrices of ever higher order. The possibility of obtaining a variety of M-matrices, for example, by rotating sub-blocks, gives a good condition for applying high-order magic squares to information security, in particular in cryptography.

Keywords: magic square, decomposition method for magic squares, arithmetic progression

Жогорку тартиптеги М-матрицаларды тургузууда негизги математикалык аппарат арифметикалык прогрессия экени ачык көрсөтүлдү. Квадраттардын константалары арифметикалык прогрессиянын мүчөлөрү катары болгон касиетинен улам чоңураак тартиптеги матрицаларды түзүү мүмкүнчүлүгүнө ээ болобуз. М-матрицалардын көп түрлөрүн алуу мүмкүнчүлүгү, мисалы, ички блокторду айлантуу жолу менен, жогорку тартиптеги магиялык квадраттын маалыматтык коопсуздукка, жекече учурда криптографияда колдонушуна жакшы шарт түзөт.

Урунттуу сөздөр: сыйкырдуу квадрат, сыйкырдуу квадраттар үчүн декомпозиция усулу, арифметикалык прогрессия.

Выявлено, что основным математическим аппаратом построения М-матриц высокого порядка, является арифметическая прогрессия. В силу свойств констант квадратов как члена арифметической прогрессии получаем возможность построения матриц все более высокого порядка. Возможность получения многообразия М-матриц, например, вращением подблоков, дает хорошее условие применения магических квадратов высокого порядка к информационной безопасности, в частности в криптографии.

Ключевые слова: магический квадрат, метод декомпозиции для магических квадратов, арифметическая прогрессия.

Various methods have been proposed for constructing magic squares (hereinafter M-matrices) [1–2]. One of the effective methods for constructing M matrices is the terrace method. However, this method has a drawback - the method works only for constructing M-matrices of odd order: $n = 2k - 1, k \in N$.

The authors implemented the algorithmization and programming of this method in electronic form. In particular, on a computer it is possible to construct M-matrices of a high order $101 \cdot 101$, even of a higher order. Note that the constant of squares of this matrix is calculated by the formula for determining the sum of the members of the arithmetic progression

$$S_{nk} = \frac{u_{1k} + u_{nk}}{2} \cdot n,$$

k is the number of the split matrix or $S_{nk+m} = S_{nk} + mk$.

To construct high-order M-matrices (both even and odd), the authors proposed a new decomposition method [1]. In this method, the main mathematical construction apparatus is the properties of an arithmetic progression. The constants of squares forming an arithmetic progression act as identifiers - operators. Note that the required set of values of square constants can also be calculated using a second-order difference equation of the form

$$u_{n+2} = 2u_{n+1} - u_n, \quad n \geq 1 \tag{1}$$

with appropriate initial conditions. According to the Euler method, the general solution of equation (1) has the form

$$u(n) = c_1 + c_2 n,$$

due to the fact that the roots of the characteristic equation $\lambda^2 - 2\lambda + 1 = 0$: $\lambda_1 = \lambda_2 = 1$ coincide. The constants of squares that form an arithmetic progression act as operators.

For clarity, let's build an M-matrix of size *9 * 9*. We apply the decomposition method. For the base we take a magic square of size *3 * 3* with spaced numbers, see Fig. 1.

8	1	6

3	5	7
4	9	2

Fig.1

We divide the given 81 digits into 9 groups: 1, 2, 3, ..., 9, 10, 11, ..., 18, 19, 20, ..., 27, 28, ..., 72, 73, 74,...81.

Further, in each group, without changing the order of the numbers, we introduce the notation: $a_1, a_2, ..., a_8, a_9$. In the first group, they are located as in the base square.

In the 2nd group $a_1 = 10$, $a_2 = 11$, $a_3 = 12$, $..., a_9 = 18$; In the 3rd group $a_1 = 19$, $a_2 = 20, a_3 = 21, ..., a_9 = 27$, ..., in the 9th group $a_1 = 73, a_2 = 74, ..., a_9 = 81$. Let us determine the constants of squares in groups. It is clear that for the base square

$$S_1 = \frac{a_1 + a_9}{2} \cdot 3 = 15.$$

a_8	a_1	a_6
a_3	a_5	a_7
a_4	a_9	a_2

Similarly: $S_2 = \frac{a_1 + a_9}{2} \cdot 3 = \frac{10 + 18}{2} \cdot 3 = 42,$

$$S_3 = \frac{19 + 27}{2} \cdot 3 = 69 = S_2 + 9 \cdot 3 = 42 + 27 \text{ and so on}$$

$$S_4 = S_3 + 27 = 96, S_5 = S_4 + 27 = 123, S_6 = 150, S_7 = 177, S_8 = 204, S_9 = 231.$$

The found constants of the squares are arranged in an M-matrix of size 3 * 3, in particular, according to the same rule, see Fig.2. Now we will consider each square constant as an identifier of the group in which it was calculated.

204	*15*	*150*
69	*123*	*177*
96	*231*	*42*

Fig. 2

Next, instead of each identifier, we will open 3 * 3 squares. For example,

$$S_1 = 15 \Leftrightarrow \begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}; S_2 = 42 \Leftrightarrow \begin{pmatrix} 17 & 10 & 15 \\ 12 & 14 & 16 \\ 13 & 18 & 11 \end{pmatrix}; \ldots ; S_8 = 204 \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} 71 & 64 & 69 \\ 66 & 68 & 70 \\ 67 & 72 & 65 \end{pmatrix} S_9 = 231 \Leftrightarrow \begin{pmatrix} 80 & 73 & 78 \\ 75 & 77 & 79 \\ 76 & 81 & 74 \end{pmatrix}$$

As a result

			8	1	6			
			3	5	7			
			4	9	2			
26	19	24						
21	23	25						
22	27	20						
			80	73	78	17	10	15
			75	77	79	12	14	16
			76	81	74	13	18	11

71	64	69	8	1	6	53	46	51
66	68	70	3	5	7	48	50	52
67	72	65	4	9	2	49	54	47
26	19	24	44	37	42	62	55	60
21	23	25	39	41	43	57	59	61
22	27	20	40	45	38	58	63	56
35	28	33	80	73	78	17	10	15
30	32	34	75	77	79	12	14	16
31	36	29	76	81	74	13	18	11

Let us calculate the constants of the square of the constructed M-matrix:

$$S = \frac{a_1 + a_{81}}{2} \cdot 9 = \frac{1+81}{2} \cdot 9 = 369.$$

Let's do a check. For example, the sum of the first row:

$$71 + 64 + 69 + 8 + 1 + 6 + 53 + 46 + 51 = 369.$$

Similarly, the sum of each row and the sum of each column, as well as the sum of the main diagonals individually, is 369. The goal is achieved.

We have built a 9*9 magic square based on the previously built one 3*3 magic square. Further, due to the constant of squares of size 3 * 3, we can get many different magic squares of size 9 * 9. Each square (block) of size 3 * 3, independently of the others, can be rotated clockwise (or counterclockwise) by $90^0; 180^0; 270^0$, then each time a new magic square of size 9 * 9 will appear. This possibility of diversity gives a good condition for the application of high-order magic squares (for example, 100*100) to information security, in particular in cryptography. This is the reason for the relevance of constructing high-order M-matrices.

Next, we will illustrate the construction of a 12th order M-matrix based on the available M-matrices of the third and fourth order. Recall that the first chosen M-matrix of the fourth order is known as the “Dürer square”.

6	7	2
1	5	9
8	3	4

Fig. 3

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

13	2	12	7
16	3	9	6
1	14	8	11
4	15	5	10

Fig. 4.

We will construct a 12th order M-matrix using the 3*4 decomposition method. Now we divide the set of numbers from 1 to 144 into 9 groups, in each group there are 16 numbers in ascending order:

1st group - 1, 2, 3,…,16,

II group - 17, 18, ..., 31, 32, and so on,

Group VIII - 113.114, ..., 127, 128,

IXth group - 129.130, ..., 143, 144.

Further, each group of numbers will be placed according to Fig. 3, counting the initial number as shifted number 1, and the last number as shifted number 16. For example, in the second group, the number 17 will be placed instead of 1; the number 18 is placed instead of 2, and so on, the last number of this group 32 is placed instead of 16. To calculate the constant of the squares of each group, we proposed the formula

$$S = \frac{n^2+1}{2}n + kn$$

k is the amount of shift. For the second group $k=16$, for the third group $k=32$, and so on. For our case $n=4$. Then the constants of the squares of the created groups form an arithmetic progression with the difference $d=16\cdot 4=64$. And so we have the first 9 members of the arithmetic progression $a_1=34$, $a_2=98$, $a_3=162$, ... , $a_9=546$, and we will place them in full accordance with the numbers in Fig. 1, that is,

$$\begin{pmatrix} a_6 & a_7 & a_2 \\ a_1 & a_5 & a_9 \\ a_8 & a_3 & a_4 \end{pmatrix} = \begin{pmatrix} 354 & 418 & 98 \\ 34 & 290 & 546 \\ 482 & 162 & 226 \end{pmatrix}$$

Note that the square constant of the last M-matrix is 870.

Further, we will consider the obtained 9 square constants as operators of expanding order. The M-matrix of the 12th order is considered as a nested 3 * 4, or rather a block matrix. Each element of the M-matrix of the third order is considered as a block and expanded, keeping its location in Fig.3. So, for example, the constant 34 will consist of M-matrices of the fourth order of the form

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

By virtue of the constants of the square, each block can be painted in an arbitrary way. We have the right to take any other M-matrix of the fourth order from the database, for example,

13	2	12	7
16	3	9	6
1	14	8	11
4	15	5	10

Fig. 5

For the constants 98 and 162, we can take the M-matrix of the fourth order built on the basis of the last Fig. 5 contained in our database:

29	18	28	23
32	19	25	22
17	30	24	27
20	31	21	26

45	34	44	39
48	35	41	38
33	30	40	43
36	47	37	42

We will continue this procedure for all constants of the squares in Fig.5. As a result, instead of each constant of the square, we write the M-matrix of our group, we get the M-matrix of the 12th order. Here is a fragment of the location instead of constants 34, 98, 162 M-matrices of the 4th order of these groups.

								29	18	28	23
								32	19	25	22
								17	30	24	27
								20	31	21	26
13	2	12	7								
16	3	9	6								
1	14	8	11								
4	15	5	10								
				45	34	44	39				
				48	35	41	38				
				33	30	40	43				
				36	47	37	42				

Finally, we obtain one form of the M-matrix of the 12th order:

93	82	92	87	109	98	108	103	29	18	28	23
96	83	89	86	112	99	105	102	32	19	25	22
81	94	88	91	97	110	104	107	17	30	24	27
84	95	85	90	100	111	101	106	20	31	21	26
13	2	12	7	77	66	76	71	141	130	140	135
16	3	9	6	80	67	73	70	144	131	137	134
1	14	8	11	65	78	72	75	129	142	136	139
4	15	5	10	68	79	69	74	132	143	133	138
125	114	124	119	45	34	44	39	61	50	60	55
128	115	121	118	48	35	41	38	64	51	57	54
113	126	120	123	33	30	40	43	49	62	56	59
116	127	117	122	36	47	37	42	52	63	53	58

It is clear that $S=\frac{a_1+a_{n^2}}{2}n=\frac{34+546}{2}3=870$. We built 9*9 (12*12) magic squares based on the previously built one 3*3 (4*4) magic square. Further, due to the constant of squares of size 3*3(4*4), we can get a lot of different magic squares of size 9*9 (12*12) from the magic square we built of size 9*9 (12*12) of each square (block) of size 3*3 (4*4) independently of the others, you can rotate clockwise (or counterclockwise) by $90^0;180^0;270^0$, then each time a new magic square of size 9*9(12*12) will appear. This possibility of obtaining a variety of matrices gives a good condition for the application of high-order magic squares to information security, in particular in cryptography. This is the reason for the relevance of constructing high-order M-matrices.

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THE PROBLEM OF OPTIMIZATION OF LIVESTOCK PRODUCTION, TAKING INTO ACCOUNT SOWING AND RENTAL AREA

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Abstract: The article developed a mathematical model and an algorithm for solving the problem, which determines the optimal size of the sown and rented area, as well as the choice of kept animal breeds on the farm without limiting production volumes according to the criterion of maximum net income.

Key words: mathematical model, rental sown area, production, net income, household, animal breeding, expenditure, solution algorithm.

Бул макалада максималдуу таза кирешенин критерийи боюнча өндүрүш көлөмүн чектебестен, ижарага алынган айдоо аянтынын оптималдуу өлчөмүн жана чарбада багылуучу малдын тукумун тандоо маселесин чечүүнүн математикалык модели жана алгоритми иштелип чыккан.

Урунттуу сөздөр: математикалык модель, арендалык эгин аянты, өндүрүш, таза киреше, экономика, мал чарбачылыгы, керектөө, чечүү алгоритми.

В статье разработана математическая модель и алгоритм решения задачи, где определяется оптимальный размер посевной и арендуемой площади, а также выбор содержимых пород животных в хозяйстве без ограничения на объемы производства продукции по критерию максимума чистого дохода.

Ключевые слова: математическая модель, арендная посевная площадь, производство, чистый доход, хозяйство, животноводство, расход, алгоритм решения.

1. Introduction

The scientific discovery of L. V. Kantorovich [1], at the intersection of mathematics and economics has become a tool for mathematical modeling and has enabled many scientists to engage in this field of science. The main theoretical aspects of economic and mathematical methods and models for solving a wide class of applied problems of economic analysis and forecasting are described in [2],[3]. In [4-14], applied economic and mathematical models of the agricultural sectors of the economy are developed and are accompanied by specific numerical examples. The authors of [14]-[16] developed economic and mathematical models for regulating prices for agricultural products and the elasticity of product quality in terms of price and costs.

In this paper, a mathematical model and an algorithm for solving the problem are developed that determine the optimal size of the sown and leased area, as well as the choice of breeds of animals kept in the farm without limiting production volumes according to the criterion of maximum net income. The developed mathematical model leads to an increase in the efficiency of the agricultural sector of the economy at the objects under study.

2. Problem statement

Let a farm having acreage of various categories (irrigated, rain-fed, etc.) in the amount of s_k, $k \in K$ planned to get the maximum income from the production of products by updating his farm with more productive cattle breeds as in [8] and expand their acreage by additionally renting acreage. The farm plans to expand its acreage based on financial capabilities. Rented acreage is limited by R_k , $k \in K$. The fee is ε_k per unit of k-th rented acreage, $k \in K$.

Suposedly, for each type of cattle breed are known its productivity and the corresponding daily feeding ration.

Besides, the yield of the fodder crop is known for each category of the sown area.

It is required to determine the optimal number of productive animal breeds and the appropriate size of the acreage to ensure maximum net income to the farm from the production and sale of livestock products, as well as the size of the acreage purchased for rent.

The mathematical model of the problem is presented below.

To find a maximum

$$L(x,y,z,v) = \sum_{h \in H} a^h z^h - \Big(\sum_{k \in K} \sum_{j \in J_0} c_{kj} x_{kj} + \sum_{h \in H} \sum_{l \in L} c_l^h y_l^h\Big) - \sum_{k \in K} \varepsilon_k v_k \tag{1}$$

having conditions

$$\sum_{j \in J_0} x_{kj} \le s_k + v_k, \quad k \in K \cup \bar{K}, \tag{2}$$

$$\sum_{k \in K} a_{kj} x_{kj} = \sum_{h \in H} \sum_{l \in L} \alpha_{jl}^h y_l^h, \quad j \in J_0, \tag{3}$$

$$\sum_{l\in L}\theta_l^h y_l^h = z^h, \quad h\in H, \tag{4}$$

$$\sum_{k\in \bar{K}}\varepsilon_k v_k \le D, \tag{5}$$

$$0\le v_k \le R_k, \ \ k\in K, \tag{6}$$

$$x_{kj} \ge 0, \quad k\in K, \ \ j\in J_0, \tag{7}$$

$$z^h \ge 0, \quad h\in H, \tag{8}$$

$$v_k \ge 0, \quad k\in K, \tag{9}$$

$$y_l^h \ge 0, \ \ h\in H, \ l\in L \ \ \text{- is integer}, \tag{10}$$

where z={ z^h: $h\in H$} , x={ x_{kj}: $k\in K$, $j\in J_0$}, y={ y_l^h :$h\in H$, $l\in L$- is integer},

$v=\{v_k: k\in K\}$.

j - is the index of the type of agricultural crop production used in the daily diet of animal feeding, $j\in J_0$;

J_0 - is the set of sown production for animal feed, $J_0=\{1,2,\ldots,n\}$;

k - is the index of the type of category of acreage in the farm, $k\in K$;

K- is the set of type of acreage categories, $K=\{1,2,\ldots,p\}$;

h - is the index of the type of livestock products produced on the farm,$h\in H$;

H - is the set of types of products animal husbandry, $H=\{1,2,\ldots,\bar{H}\}$;

l - is the index of the type of animal breed in the farm, $l\in L$;

L- is the types of types of animal breeds, $L=\{1,2,\ldots,\bar{L}\}$.

Known parameters:

s_k - is the size of the sown area k -th categories in the household, $k\in K$;

a_{kj} - is yield j - th type of sown on k -th categories of acreage of the farm, $k\in K$, $j\in J_0$;

α_{jl}^h - is an annual demand of j-th type of sown production per animal of l-th breeds in production h-th types of products, where

$$\alpha_{jl}^h = \beta_{jl}^h\gamma_{jl}^h, \quad j\in J_0, \ \ l\in L, \ \ h\in H; \tag{11}$$

β_{jl}^h - is a share of j -th sown production in the daily diet per animal of l-th breeds on the farm for production h- th types of products, $j\in J_0$, $l\in L$, $h\in H$;

γ_{jl}^{h} - is amount of days in the diet of feeding with plant products of j -th type for l-th animal breeds in production of h-th type of product, $j \in J_0$, $l \in L$, $h \in H$;

θ_l^h - is the volume of products of the h-th type received by the farm from one animal of l-th breed, $l \in L$, $h \in H$;

ε_k - is the fee per unit of k-th rented acreage, $k \in K$;

R_k – is the maximum of possible size of the k-th type of rented acreage, $k \in K$;

D - is the financial ability of the farm to pay for rented acreage;

a^h – is sale price of h-th types of livestock products on the farm, $h \in H$;

c_{kj} – are costs per unit size of the k-th category of acreage for the j-th type of culture, $j \in J_0$, $k \in K$;

c_l^h - is annual consumption per animal of the l-th breed in the production of the h-th type of livestock products, $h \in H$, $l \in L$.

Variables to find:

x_{kj} – is size from the k -th category of the sown area allocated for the j-th type of crop, $j \in J_0$, $k \in K$;

y_l^h - is the amount of animals of the l-th breed in the farm for the production of the h-th type of product, $h \in H$, $l \in L$;

v_k – is size of the k-th category of the leased sown area, $k \in K$;

z^h – is volume of sold livestock products of the h-th type, $h \in H$.

Target function (1) determines the maximum net income received by the farm from the production of various categories of sown and rented areas and the sale of livestock products;

Limit (2) shows that for the cultivation of forage crops of all types, the available acreage and rented acreage of various categories are occupied;

Limit (3) show that the volume of agricultural products of each type produced for feed must be equal to the volume of the farm's needs for internal needs (for feed);

Limit (4) requires that the volume of livestock products of each type must be sold;

Limit (5) shows that the size of the leased acreage of various categories does not exceed the financial capacity of the farm;

Limit (6) shows that the sizes of leased acreage of various categories are limited to the maximum possible size of leased acreage;

Limits (7) (8), (9) require non-negativity of variables;

Limit (10) requires that the value of variables must be an integer.

Table 1

Recording the condition of the problem (1)-(10) in the form of a table

x_{11}	x_{12}	…	x_{1n}	x_{21}	x_{22}	…	x_{2n}	…	x_{p1}	x_{p2}	…	x_{pn}	z^1	z^2	…	z^h	y_1^1	y_2^1	…
1	1	…	1																
				1	1	…	1												
								…											…
									1	1	…	1							
a_{11}				a_{21}					a_{p1}								$-\alpha_{11}^1$	$-\alpha_{12}^1$	…
	a_{12}				a_{22}					a_{p2}							$-\alpha_{21}^1$	$-\alpha_{22}^1$	…
		…				…		…			…						…	…	…
			a_{1n}				a_{2n}					a_{pn}					$-\alpha_{n1}^1$	$-\alpha_{n2}^1$	…
													-1				θ_1^1	θ_2^1	…
														-1					
															…				…
																-1			
c_{11}	c_{12}	…	c_{1n}	c_{21}	c_{22}	…	c_{2n}	…	c_{p1}	c_{p2}	…	c_{pn}	a^1	a^2	…	a^h	c_1^1	c_2^1	…

Table 1 continued

…	y_l^1	y_1^2	y_2^2	…	y_l^2	…	y_1^h	y_2^h	…	y_l^h	v_1	v_2	…	v_k		
											-1				≤	S_1
												-1			≤	S_2
													…		…	…
														-1	≤	S_p
…	$-\alpha_{1l}^1$	$-\alpha_{11}^2$	$-\alpha_{12}^2$	…	$-\alpha_{1l}^2$	…	$-\alpha_{11}^h$	$-\alpha_{12}^h$	…	$-\alpha_{1l}^h$					=	0
…	$-\alpha_{2l}^1$	$-\alpha_{21}^2$	$-\alpha_{22}^2$	…	$-\alpha_{2l}^2$	…	$-\alpha_{21}^h$	$-\alpha_{22}^h$	…	$-\alpha_{2l}^h$					=	0
…	…	…	…	…	…	…	…	…	…	…					…	…
…	$-\alpha_{nl}^1$	$-\alpha_{n1}^2$	$-\alpha_{n2}^2$	…	$-\alpha_{nl}^2$	…	$-\alpha_{n1}^h$	$-\alpha_{n2}^h$	…	$-\alpha_{nl}^h$					=	0
…	θ_l^1														=	0

		θ_1^2	θ_2^2	$\cdots$	θ_l^2										=	0
						$\cdots$									…	…
							θ_1^h	θ_2^h	$\cdots$	θ_l^h					=	0
											ε_1	ε_2	…	ε_k	≤	D
											1				≤	R_1
												1			≤	R_2
													…		…	
														1	≤	R_k
$\cdots$	c_l^1	c_1^2	c_2^2	$\cdots$	c_l^2	$\cdots$	c_1^h	c_2^h	$\cdots$	c_l^h	ε_1	ε_2	…	ε_k	→	max

The algorithm for solving the problem. The algorithm for solving the problem (1)-(10) differs from the algorithm described in [10, 11] with minor changes. As in [10], calculations begin with the calculation of the parameter values α_{jl}^h, $j \in J_0$, $l \in L$, $h \in H$ according to the formula (11).

Using parameter values a_{kj}, c_{kj}, s_k, ε_k, R_k, $k \in K$, $j \in J_0$ и θ_l^h, c_l^h, a^h, $h \in H$, $l \in L$, D, a numerical model of the problem is formulated according to (1)-(10).

From the solution of the problem, the number of cattle of the farm is determined $y=\{ y_l^h :, h \in H, l \in L\}$, the appropriate size of the acreage allocated for fodder crops $x=\{ x_{kj}: k \in K, j \in J_0$ and the size of the acreage purchased for rent v_k: $k \in K$, as well as the volume of livestock products sold $z=\{z^h: h \in H\}$, allowing to provide the maximum net income to the household. The solution algorithm ends.

Example. Let a farm engaged in the production of livestock products (milk and meat), having a financial resource in the amount of 1000000 soms and having sown areas in the amount of S = 366 ha, of which irrigated S_1 = 280 ha, and rainfed S_2 = 86 ha, planned to receive the maximum income from production by updating its farm with more productive breeds of cattle and increasing acreage by additionally renting acreage. He plans to increase the area under crops, based on his financial capabilities. Leased crop areas are limited by R_k, k=1,2, i.e. $R_k=\{R_1,R_2\}=\{300,150\}$. We also know the payment ε_k per unit of the k-th leased area, k=1,2, i.e. $\varepsilon_k=\{\varepsilon_1,\varepsilon_2\}=\{2000,1500\}$.

The farm has a choice of animals of productive breed for the production of milk and meat with the appropriate daily feeding ration [8]. Two breeds of dairy cows and two breeds of beef cows. For milk production:

-the first type of breed - dairy cows with a milk yield of 3600 kg with a daily feeding ration (Table 2);

- the second type of breed - dairy cows with a milk yield of 4500 kg with a daily feeding ration (Table 3);

Similarly, the first and second types of breeds for the production of meat - bulls (heifers) with a diet of feeding with a live weight of 300 kg, respectively and 450 kg meat (Tables 4 and 5).

Table 2

Daily feeding ration for dairy cows of the first breed with a milk yield of 3600 kg

	Feed name		Daily ration kg (1 animal)		Fodder units	common unit	Total for 1 animal	number of days
1	Medick (dry fodder)		4		0,5	2,0	720,0	180
2	chaff	wheat	3	1	0,2	0,6	180.0	180
		barley		2			360.0	
3	haylage		6		0,3	1,8	1080,0	180
4	The concen-tration of the feed	wheat	2,4	0,3	1	2,4	109,5	365
		barley		1,5			547,5	
		corn		0,6			219,0	
5	Silage(corn)		10		0,3	3,0	1800,0	180
6	Mineral feed		0,010		-	-	-	365
7	salt		0,030		-	-	-	365
8	Pasture forage		40		-	-	7200,0	180
	Green feed							
	Total		-		-	9,8	-	-

Table 3

Daily ration of feeding milk cows of the second breed with milk yield 4500 kg of milk

	The name of the fodder		**Daily ration (1 animal)**		**Number of days**	**For a year (1animal)**
1.	Medick (dry fodder)		10 kg		180	1800 kg
2.	chaff	Wheat	3 kg	1 kg	180	180 kg
		Barley		2 kg		360 kg
3.	Haylage		8 kg		180	1440 kg
4.	The concen-tration of the feed	Wheat	3 kg	2 kg	365	730 kg
		Barley		0,5 kg		182,5 kg
		seed (corn)		0,5 kg		182,5 kg

5.	Silage (corn)	12 kg	180	2160 kg
6.	Mineral feed	-	365	3,6 kg
7.	salt	-	365	10,8 kg
8.	Pasture forage / Green fodder	50 kg	180	9000 kg

Table 4

Daily ration for feeding cattle (bulls, heifers) with live weight 300 kg

	The name of the fodder		**Daily ration (1 animal), kg**		**Fodder unit**	**General unit**	**Total for 1 animal**	**Days qty**
1.	Medick (dry fodder)		3		0,5	1,5	540,0	180
2.	chaff	Wheat	2	0,5	0,2	0,4	90,0	180
		Barley		1,5			270,0	
3.	Hayalage		6		0,3	1,8	1080,0	180
4.	Conc. feed	Barley	1,5	0,5	1	1,5	182,5	365
		Wheat		0,5			182,5	
		Seed (corn)		0,5			182,5	
5.	Silage (corn)		5,0		0,3	1,5	900,0	180
6.	Mineral feed		0,010		-	-	-	365
7.	Salt		0,030		-	-	-	365
8.	Pasture forage / Green fodder		30		-	-	5400,0	180
	Total		-		-	6,7	-	-

Table 5

Daily ration for feeding cattle (bulls, heifers) for meatwith a live weight of 450 kg

	The name of the fodder		**Daily ration (1 head)**		**Days qty**	**For a year (1 head)**
1.	Medick (dry fodder)		5 kg		365	1825 kg
2.	chaff		1 kg		365	365 kg
3.	Hayalage		10 kg		180	1800 kg
4.	Conc. of the feed	Barley	3 kg	1 kg	365	365 kg
		seed (corn)		1 kg		365 kg
		Wheat		1 kg		365 kg
5.	Silage (corn)		10 kg		180	1800 kg
6.	Mineral feed		-		365	-
7.	Salt		-		365	-
8.	Pasture forage / Green fodder		30 kg		180	5400 kg

In addition, the following are known: - the selling price of a unit volume of milk $a^1 = 25$ soms, meat $a^2 = 140$ soms;

- crop yield on irrigated fields (I) and rainfed (II), included in the feeding ration, a_{kj}, k=1,2, j=1,2,…,7 (Table 6).

Table 6.

	wheat	barley	alfalfa hay	hayalage	Green fodder	Silage (corn)	Grain (corn)
	1	2	3	4	5	6	7
I	2070.0	1962.2	2380.0	6281.0	5730.0	12340.0	20280.0
II	1500.0	0	1700.0	0	0	0	0

-the are expenses for growing agricultural crops per unit of size (I) and (II) fields, $|c_{kj}|_{2,7}$, Table 7.

Table 7.

	1	2	3	4	5	6	7
I	2279.0	1096.0	2618.0	5071.0	9225.0	13574.0	7743.0
II	2000.0	1000.0	2000.0	5000.0	9000.0	13000.0	7000.0

Also known is the cost of keeping one cattle of each breed in the production of milk and meat, respectively $c_{l=1}^{h=1}$=50040.0 soms, $c_{l=2}^{h=1}$=55080.0 soms, $c_{l=1}^{h=2}$ =18030.0 soms, $c_{l=2}^{h=2}$=25020.0 soms.

It is required to determine the optimal size of the sown area of a different category, the size of the leased sown area, based on the financial capacity of the farm, and the corresponding number of productive animal breeds, allowing to ensure the maximum net income for the farm from the production and sale of livestock products.

For mathematical formalization of the problem, let's determine the annual feed requirement per animal of each type of breed in the production of products, α_{lj}^{h}, $j \in J_0$, $l \in L$, $h \in H$.

Using the daily feeding ration, we determine the annual need for each type of agricultural product included in the feed for one dairy cow with a milk yield of 3600 kg and one dairy cow with a milk yield of 4500 kg. We will also determine the annual need for each type of agricultural product for feed for one bull and a heifer of the first and second type of breed for meat (see Table 8).

Table 8.**Annual feed requirement per animal, depending from breed and productivity**

The name of the fodder	Feed requirement per dairy cow		Feed requirement per cow for meat	
	1 type of breed with 3600 kg milk yield	2 type breed with 4500 kg milk yield	1 breed with a live weight of 300 kg	2 species of live weight breed 450 kg
1.wheat	289,5	362,5	272,5	365,0
2.barley	907,5	1090,0	452,5	730,0
3. perennial grass 3.1.Medick (dry fodder)	720,0	1800,0	540,0	1825,0
3.2. haylage	1080,0	1440,0	1080,0	1800,0
3.3. Green feed	7200,0	9000,0	5400,0	5400,0
4. Corn 4.1.silage	1800,0	2160,0	900,0	1800,0
4.2.seed	219,0	182,5	182,5	365,0

We formulate a numerical model of the problem.

Find a maximum

$$L(x,y,z,v) = 25z^1 + 140z^2 - (2279.0x_{11} + 1096.0x_{12} + 2618.0x_{13} + 5071.0x_{14} + 9225.0x_{15} +$$

$$+13574.0x_{16} + 2000.0x_{21} + 1000.0x_{22} + 2000.0x_{23} + 5000.0x_{24} + 9000.0x_{25} +$$

$$+13000.0x_{26} + 50040.0\, y_1^1 + 55080.0\, y_2^1 + 18030.0\, y_1^2 + 25020.0\, y_2^2) -$$

$$- \quad 2000v_1 - 1500\, v_2 \tag{12}$$

under conditions

$$\sum_{j=1}^{7} x_{1j} \le 280 + v_1, \qquad \sum_{j=1}^{7} x_{2j} \le 86 + v_2, \tag{13}$$

$$2070,0x_{11} + 1500,0x_{21} = 289,5\, y_1^1 + 362,5\, y_2^1 + 272,5\, y_1^2 + 365,0\, y_2^2,$$

$$1962,0x_{12} + 0x_{22} = 907,5\, y_1^1 + 1090,0\, y_2^1 + 452,5\, y_1^2 + 730,0\, y_2^2,$$

$$2380,0x_{13} + 1700,0x_{23} = 720,0\, y_1^1 + 1800,0\, y_2^1 + 540,0\, y_1^2 + 1825,0\, y_2^2,$$

$$6281,0x_{14} + 0x_{24} = 1080,0\, y_1^1 + 1440,0\, y_2^1 + 1080,0\, y_1^2 + 1800,0\, y_2^2,$$

$$5730,0x_{15} + 0x_{25} = 7200,0\, y_1^1 + 9000,0\, y_2^1 + 5400,0\, y_1^2 + 5400,0\, y_2^2,$$

$$12340,0x_{16} + 0x_{26} = 1800,0\, y_1^1 + 2160,0\, y_2^1 + 900,0\, y_1^2 + 1800,0\, y_2^2, \tag{14}$$

$$3600\, y_1^1 + 4500\, y_2^1 \ge z^1, \qquad 300\, y_1^2 + 450\, y_2^2 \ge z^2, \tag{15}$$

$$2000\, v_1 + 1500\, v_2 \le 1000000, \tag{16}$$

$$v_1 \le 300, \quad v_2 \le 150, \tag{18}$$

$$x_{kj} \ge 0, \quad k=1,2, \quad j=1,2,\dots,7, \tag{19}$$

$$z^h \geq 0, \quad h \in H, \tag{20}$$

$$v_k \geq 0, \quad k \in K, \tag{21}$$

$$y_l^h \geq 0, \quad h \in H, \quad l \in L \text{ - is integer,} \tag{22}$$

We write the numerical model of problem (12) - (22) in the form of a table 9.

Table 9. **Presentation of the conditions of problem (12) - (22) in the form of a table**

Z_1	Z_2	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{21}	x_{22}	x_{23}
		1	1	1	1	1	1			
								1	1	1
		2070.0						1500.0		
			1962.0						0	
				2380.0						1700.0
					6281.0					
						5730.0				
							12340.0			
-1										
	-1									
25,0	140,0	- 2279.0	- 1096.0	- 2618.0	- 5071.0	- 9225.0	- 13574.0	- 2000.0	- 1000.0	-2000.0

Continuation of table 9.

x_{24}	x_{25}	x_{26}	y_1^1	y_2^1	y_1^2	y_2^2	v_1	v_2		
							-1		≤	280
1	1	1						-1	≤	86
			-289,5	-362,5	-272,5	-365,0			=	0
			-907,5	-1090,0	-452,5	-730,0			=	0
			-720,0	-1800,0	-540,0	-1825,0			=	0
0			-1080,0	-1440,0	-1080,0	-1800,0			=	0
	0		-7200,0	-9000,0	-5400,0	-5400,0			=	0
		0	-1800,0	-2160,0	-900,0	-1800,0			=	0
			3600,0	4500,0					=	0
					300,0	450,0			=	0

							1		≤	300
								1	≤	100
							2000	1500	≤	1000000
-5000.0	-9000,0	-13000,0	-50040,0	-55080,0	-18030,0	-25020,0	-2000	-1500	→	max

Let's solve the problem (12)-(22) using the method used in [10].

We will get the optimal plan for the distribution of sown areas for fodder crops x={ $x_{11}=32.77$; $x_{12}=120.13$; $x_{13}= 0$; $x_{14}=49.57$; $x_{15}=339.65$; $x_{16}=37.85$; $x_{21}=7.03$, $x_{22}=0$, $x_{23}=228.97$, $x_{24}=0$, $x_{25}=0$, $x_{26}=0$},

the composition of the kept breeds of cattle in the farm of dairy and meat direction y={ $y_1^1=0$; $y_2^1=216$; $y_1^2=0$; $y_2^2=1$}, the volume of products sold by the farm z={ $z_1=972.0$ tons}, and volumes of leased acreage v={$v_1=300$, $v_2=150$}, as well as the amount of net income of the farm $L(x,y,z,z)=7015069.36$ soms.

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AN ECONOMETRIC MODEL FOR ASSESSING STRUCTURAL SHIFTS

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This article is devoted to the actual problem of developing models of structural imbalances formed in the national economy of the Kyrgyz Republic. In the article, structural imbalances are considered from the standpoint of one of the fundamental and urgent problems of sustainable growth of the national economy. At the same time, in accordance with the formulated goal, the following task is solved: to reveal the specifics of structural imbalances in the national economy of the country as the main factor of sustainable economic growth.

Keywords: Structural imbalances, sustainable economic growth, econometric model, production function.

Бул статья Кыргыз Республикасынын улуттук экономиксындагы түзүлүштүк диспропорциянын пайда болушунун себептерин изилдөөгө арналган моделдерди иштеп чыгуу актуалдуу проблемасына арналган. Статьяда түзүлүштүк диспропорцияны изилдөөдө фундаменталдуу жана актуалдуу болгон улуттук экономиканын туруктуу өнүгүшүү позициясынан боюнча жүргүзүлгөн. Коюлган максатка жараша өлкөнүн эл чарбасындагы түзүлүштүк диспропорция экономикалык өсүштүн эң негизги фактору катары каралган маселе чечилет.

*Урунтуу сөздөр:*Түзүлүштүк диспропорция, туруктуу экономикалык өсүш, эконометрикалык модель, өндүрүш функциясы.

Данная статья посвящена актуальной проблеме разработки моделей структурных диспропорций, формирующихся в национальной экономике Кыргызской Республики. В статье структурные диспропорции рассматриваются с позиций одной из фундаментальных и актуальных проблем устойчивого роста национальной экономики. При этом в соответствии с сформулированной целью решается следующая задача: выявить специфику структурных диспропорций в народном хозяйстве страны как основного фактора устойчивого экономического роста.

Ключевые слова: Структурные диспропорции, устойчивый экономический рост, эконометрическая модель, производственная функция.

1. Introduction

At the present stage, one of the most difficult problems of economic reforms in the CIS countries, including in Kyrgyzstan, is that they face the task of overcoming severe structural imbalances in the economy, inherited from the administrative-command system of the past.

Modeling macroeconomic imbalances is not easy. By their very nature, imbalances and their possible drivers are likely to be identified simultaneously, leading to problems of endogeneity in empirical analysis. Under the conditions of international economic integration (EAEU), the existing structural disproportions in the economy not only hinder the realization of economic potential, but also increase the vulnerability of the economy to external influences, and also reduce the standard of living of the country's population. Identification and elimination of structural disproportions is an objective need of the national economy, necessary for entering the trajectory of sustainable, stable development of the economy.

2. Research methods

The economy of the Kyrgyz Republic as a macroeconomic system is dynamic and complex. This is due to the fact that, for sustainable growth, it is necessary to provide a balanced supply of labor, material and financial resources. Therefore, solving the problem of sustainable economic development requires the use of econometric tools and methods of econometric modeling.

We have developed and applied a three-sector model based on *CES* - the production function (CES - Constant Elasticity of Substitution) developed by K. Arrow, G. Chineri, B. Michnas and R. Solow in 1961 [2].

Indicators of population growth rates, rates of technical progress, rates of depreciation of capital or savings rates will be considered constant. In the process, we can accurately assess the dynamics of the model so that we can determine what will happen if these parameters change.

The three-sector model divides the economic sectors into three sectors of activity: extraction of raw materials (primary), processing (secondary) and services (tertiary). According to the model, the main focus of the economy is shifting from the primary to the secondary and, finally, to the tertiary sector.

Widely used linear models to describe complex phenomena may, in some cases, give only rough first approximations. When applying more complex models, for example, non-linear regression equations, there are two cases: a) non-linear regression equations are "intrinsically linear", i.e. they can be converted to a linear form using a change of variables and using a logarithm; b) the regression equations are "intrinsically non-linear", i.e. they can be converted to a linear form with respect to unknown parameters. The *CES* production function is of the second type, i.e. is non-linear with respect to its parameters, which makes their calculation difficult, not allowing, in particular, to apply the usual least squares method and causes great difficulties when working with it.

Among the production functions (PF) used in modeling, the most popular in economic research and are interconnected: a function with constant elasticity of substitution or *CES* - function, Cobb-Douglas, linear and Leontief functions.

The *CES* production function is a special form of the neoclassical production function and in this approach the production technology uses a constant percentage change in the proportions of factors (capital and labor), and has the form:

$$Y = f(K,L) = A(bK^{-\rho} + (1-b)L^{-\rho})^{-\delta/\rho}, A > 0, \rho = \frac{1-\sigma}{\sigma}, 0 < \sigma < 1, 0 < b < 1.$$

where σ – the elasticity of the replacement of the labor factor by the capital cost factor: σ-const; δ – the index of homogeneity of the production function; A,b – constants, the values of which are determined by the method of least squares on the basis of retrospective macroeconomic data of the economic system.

A special case of the *CES* - function at σ=1 and δ=1 is the production function of Cobb - Douglas:

$$Y = f(K,L) = AK^{\alpha}L^{\beta}, A > 0, \alpha + \beta = 1, \qquad \alpha > 0, \beta > 0.$$

Let us assume that each sector has fixed production assets on its balance sheet. Investments and labor resources can move freely between sectors. Then the following conditions can be accepted:

- The technological structure is considered constant and is set using linearly homogeneous neoclassical production functions

$$Y_i = F(K_i L_i), i = 0,1,2.$$

- The total number of employed L (in the manufacturing sector) changes with a constant growth rate *v*. Then *v* in discrete time has the form (*t* is the number of the year).

$$v = \frac{L(t+1) - L(0)}{L(t)}$$

the form of the differential equation in the transition to continuous time takes

$\frac{dL}{dt} = vL,\ L(0) = L^0$ its solution is $L = L^0 e^{vt}$.

- There is no investment lag.

- Depreciation coefficients of fixed production assets and direct material costs sectors are constant.

- Foreign trade is not considered, thus the economy is closed.

- Time changes continuously.

From assumptions (3, 4) it follows that the change over the year of the BPF consists of two parts: depreciation *(-K)* and growth due to gross capital investments *(I)*, that is

$$K_i(t+1) - K_i(t) = -\mu_i K_i + I_i(t), \qquad i = 0,1,2.$$

or in continuous form:

$K_i(t+\Delta t) - K_i(t) = [-\mu_i K_i(t) + I_i(t)]\Delta t$ at $\Delta t \to 0$.

Thus, we have a differential equation for the OPF of sectors:

$$\frac{dK_i}{dt} = \mu_i K_i + I_i(t),\ \ K(0) = K^0.$$

Three-sector model of the economy:

$$L = L^0 e^{vt};$$

$L = L_0 + L_1 + L_2$ – distribution of employed by sectors.

$\frac{dK_i}{dt} = \mu_i K_i + I_i(t),\ \ K(0) = K^0$ – dynamics of funds by sectors.

$X_i = F(K_i L_i), i = 0,1,2$ – output by sectors.

$Y_1 = I_0 + I_1 + I_2$ – distribution of production of the fund-creating sector.

$Y_2 = \alpha_0 X_0 + \alpha_1 X_1 + \alpha_2 X_2 -$ distribution of production of the material sector.

It is argued that, at the macro level, the economic system also strives to include as much functioning capital in production and to employ as much labor force in order to provide a maximum of $Y = Y_0 + Y_1 + Y_2$.

To study structural disproportions, we have developed a three-sector model of the economy. The three-sector model divides economic sectors into three sectors of activity: extraction of raw materials (primary), manufacturing (secondary) and services (tertiary). According to the idea of the three-sector model, the main focus of the economy is shifting from the primary to the secondary and, finally, to the tertiary sector. It is assumed that each sector has fixed production assets on its balance sheet, investments and labor resources can move freely between sectors. Then the following conditions can be accepted:

–The technological structure is considered constant and is set using linearly homogeneous (by parameters) neoclassical production functions

$$X_i = F(K_i, L_i) \;\; i = 1,2,3.$$

– The total number of employed L (in the manufacturing sector) changes with a constant growth rate.

Received the following PF sectors

To find the parameters of a three-sector model, general minimization algorithms can be used to minimize functions of many variables of a general type. The specific form of the functions of this model makes it possible to construct simpler minimization algorithms. There are various methods for minimizing *CES* - functions. To find the parameters, we used the Marquardat method.

The Marquardat method is a compromise between the Gauss-Newton method and the gradient method. This method actually performs an optimal interpolation between the linearization method and the gradient method.

When using iterative methods, special attention should be paid to the choice of the initial approximation. To select the initial approximations, a method is usually used that allows, having a priori fixed the values of some function parameters, to obtain estimates of the remaining parameters. This method was used by us to obtain preliminary estimates of the parameters of the production *CES* - function, followed by its evaluation by the iterative method of Marquardt.

We carried out several variants of calculations for sectors of the economy. The resulting parameter estimates and alignment characteristics are shown in Table 1. When estimating the parameters, we used the results of preliminary analyzes of the initial information and excluded from the series the points containing the most obvious observational errors. From several variants of calculations carried out for all sectors, production functions with the best leveling characteristics were selected.

						$\boldsymbol{\sigma^2}$		*W*	$\boldsymbol{R^2}$	*F-statistic*
	1									10
	GDP total	,968	,397	,168	,31	,05	,17	,145	,924	170 6,3
	Primary (mining of raw materials)	,904	,234	,72	,721	,112	,07	,224	,916	173 4,4
	Secondary (processing)	,052	,733	,754	,961	,07	,38	,838	,961	148, 8
	Tertiary (services)	,644	,184	,115	,428	,09	,05	,032	,99	464, 64

Table 1. Characteristics of *CES* - production function.

The national economy as a whole, the resulting production function has the following form: $Y = 0{,}832(0{,}332K^{-1{,}124} + 0{,}678L^{-1{,}124})^{-\frac{1{,}22}{1{,}124}}$

The alignment characteristics show that the obtained parameters are significant. The standard deviation of the level is minimal: $\mathrm{y}^{\mathbf{2}} = 0{,}05$; squared variation $v = 2{,}32\%$ insignificant. Estimates of the coefficient of determination are quite high $R^2 = 0{,}903$. F is equal to 1801.6.

Compared with other sectors, at the level of the national economy as a whole, the leveling performance is much better. The values of the coefficients of variation allow us to state that the influence of factors on output is relatively stable.

Elasticity coefficient $б(\boldsymbol{x}) = \mathbf{0,132}$; indicates that the economy as a whole is characterized by a labor-saving trend of technological change, i.e. a decrease in elasticity for fixed production assets and an increase in elasticity for the cost of living labor. From the point of view of *CES* - the production function, the decrease in the growth rate of funds and the decrease in the return on assets $Y/_{K}$ can be explained by the decrease in the growth rate of living labor with an insufficiently high level of elasticity (0,365).

3. Conclusions

The principle of sustainable economic growth should offer an absolutely clear strategy for the implementation of structural reforms, in order to increase productivity and in the framework of managing imbalances.

The government should not ignore the existence of macroeconomic territorial and sectoral imbalances, and in the future it is necessary to agree at the state level and implement a number of reforms in the field of managing economic imbalances through investment policy.

The government needs to consider this issue together with key bodies in the state structure and revise its strategy at the government level in order to attract investment, improve technical support in the first place in those sectors where there is a clear disproportion, and to improve the business climate in the whole country.

The work was carried out within the framework of the project of the Institute of Mathematics of the National Academy of Sciences of the Kyrgyz Republic.

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MODELING OF THE MAIN PARAMETERS OF URBAN PASSENGER TRANSPORT

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The article analyzes the main factors influencing the organization of the effective functioning of the work of urban passenger transport. The analysis shows that the type of vehicles, the configuration of the route network greatly affects the overall costs of transport systems and the costs of passengers. In this regard, an optimization mathematical model for choosing vehicles of the required capacity is considered in order to ensure the most complete satisfaction of the needs of passengers in transportation at the lowest cost. The model assumes that the transport network has a limited capacity.

Key words: mathematical model, rolling stock, route, urban passenger transport system.

Статьяда шаардык пассажирдик транспорттун эффективтүү иштешин уюштурууга таасир берүүчү негизги факторлор анализденет. Анализ көргөзгөндөй, жалпы транспорттук чыгашага жана жүргүнчүлөрдүн чышасына транспорттун түрлөрү, маршруттук түйүндүн конфигурасиясы олутту таасир берет. Ошондуктан статьяда жүргүнчүлөрдүн талабын толук канатандыруу үчүн, керектүү көлөмдөгү транспорттун түрлөрүн тандоо үчүн оптималдаштыруучу математикалык модели каралган. Моделде транспорттук түйүн чектелген өткөрүү мүмкүнчүлүгүнө ээ деп сунушталат.

Урунттуу сөздөр: математикалык модель, кыймылдуу состав, маршрут, шаардык жүргүнчүлөрдү ташуучу транспорт системасы.

В статье анализируется основные факторы, влияющие на организацию эффективного функционирования работы городского пассажирского транспорта. Анализ показывает, что, вид транспортных средств, конфигурация маршрутной сети сильно влияет на общие затраты транспортных систем и на затраты пассажиров. В связи этим, рассмотрены оптимизационная математическая модель выбора транспортных средств необходимой вместимости, с целью обеспечения наиболее полного удовлетворения потребности пассажиров в перевозках с наименьшими затратами. В модели предполагается, что, транспортная сеть имеет ограниченную пропускную способность.

Ключевые слова: математическая модель, подвижной состав, маршрут, городская пассажирская транспортная система.

Currently, Bishkek is subject to a large number of traffic congestion, traffic accidents, as well as air pollution from exhaust gases. The traffic management system was introduced during the Soviet Union and is still functioning, but its effectiveness is very low due to outdated equipment and heavy traffic. Public urban transport in Bishkek consists of trolleybuses, buses and minibuses. Trolleybuses and buses are operated by public agencies, while minibuses are operated by private bus companies.

The main problems of the public urban transport sector are – reconsideration of the role of trolleybuses, buses and minibuses and proper distribution of modes of transport, provided that the indicator of road load by the corresponding types of vehicles is improved; – optimization of the number of buses by introducing large buses (instead of minibuses) and increasing the profitability of each bus and coordinating the routes and intervals of urban trolleybuses and buses.

The operation of urban passenger transport depends on many different factors, such as the structure of the road network, the distance between stops, pricing policies, fares, intervals and type of rolling stock.

For our study, we developed models with respect to two variables – the interval of movement and the type of rolling stock. Assume that the other factors are difficult to quantify because they already exist and relatively meet the demand criteria. Therefore, their influences will be considered fixed.

Consider a two-level optimization mathematical model for choosing the optimal size of the rolling stock. At the top level, the function represents the costs of the population and the costs of the managing company. At the lower level, it includes a model for the distribution of passenger flows.

Variables are the intervals of movement of each route, where n is the number of routes in the transport network. The value n = 1 if the k – bus type is used on route I, and n = 0 otherwise.

The costs used in this model consist of passenger costs (ZP) and operating costs (ZE). Passenger costs are obtained by simulation and depend on variables and are calculated using the following formula:

$$ZP = \alpha_1 BP + \alpha_2 BO + \alpha_3 BD + \alpha_4 BPE. \quad (1)$$

where

BP – the total time of approach to the stopping point;

BO – the total waiting time;

BD – the total time of movement;

BPE – total time spent on the transfer;

α_1 – the value of the time of approach to the stopping point;

α_2 – waiting time value;

α_3 – the value of the movement time;

α_4 – the value of the time spent on the transfer.

The total cost is (kilometers):

$$CK = \sum_i \sum_k L_i f_i CK_k \delta_{k,i}. \quad (2)$$

where

L_i – length of route *I*;

f_i – traffic interval on route *I*;

CK_k – the unit cost of a kilometer of travel on a bus of type *k*;

$\delta_{k,i}$ – the variable is assigned the value "1" if the type of bus is assigned to the route, and the value "0" in other cases.

The cost of waiting for the rolling stock at the stopping point:

$$CR = TG_k \sum_i \sum_k CR_k \delta_{k,i} Y_i. \quad (3)$$

where

TG_k – the average time of boarding and disembarking passengers, for a bus of type *k*;

CR_k – costs per passenger per hour while waiting for rolling stock, on a certain type of bus;

Y_i – the demand for a trip (obtained using transport network modeling).

$$TG_k = \beta_0 + max_j\{\beta_s NS_j + \beta_B N\beta_j\} \quad (4)$$

where

$NS_j(N\beta_j)$ – the number of incoming/outgoing passengers who used door j at the stop;

β_0, β_s, β_B – are parameters.

If the stopping point is overloaded, then the movement of urban passenger transport becomes disorganized and the time increases (β_0), and the time limit before the arrival of the next rolling stock (β_s) may increase. Similarly, the time limit to get off the bus (β_B) increases if the bus is full and it takes longer for passengers to move through the cabin. Staff costs are taken as:

$$CP = C_p \sum_i f_i. \tag{5}$$

where

C_p – the cost of transporting passengers per hour.

Fixed costs for buses are calculated using the following formula:

$$CF = \sum_i \sum_k f_i CF_k \delta_{k,i}. \tag{6}$$

Based on the above cost structure, the optimization problem for the top level is as follows:

$$minZ = \alpha_1 BP + \alpha_2 BO + \alpha_3 BD + \alpha_4 BPE + \sum_i \sum_k L_i f_i CK_k \delta_{k,i} +$$
$$+t_{sb} \sum_i \sum_k CR_k \delta_{k,i} Y_i + C_p \sum_i f_i + C_p \sum_i f_i + \sum_i \sum_k f_i CF_k \delta_{k,i}; \tag{7}$$

$\delta_{k,i} \in (0,1)$;

$\sum_k \delta_{k,i} = 1\ \forall i$;

$\sum_i f_i = \sum_i \sum_k \frac{Y_i \delta_{k,i}}{K_k O_k}$.

The first constraint is determined with respect to the characteristics of binary variables $\delta_{k,i}$. The second limitation is that only one type of bus can be assigned to each route. The third constraint is the satisfaction of demand depending on the capacity of various types of buses, where K_k is the capacity of bus type k, and O_k is the bus load factor, which varies depending on the filling and takes a value from 0 to 1.

Destination Model

The lower level is optimized by applying the public transport destination model. The equilibrium conditions established for the problem under consideration can be formulated using the variational inequality of the following form:

$$c(v^*) * (v^* - v) \leq 0, \forall V \in \Omega \tag{8}$$

where

c is the cost vector on the route sections;

V is any admissible vector for flows on route segments;

$\{V_s\}$ and V^* represent an equilibrium solution with respect to flows on route sections.

The method is often used to find a solution in cases where the diagonalization algorithm is applied, where the cost function can be found at each iteration. Alternatively, the method can be used in the direct solution of the equilibrium assignment problem.

Therefore, to solve an equilibrium model in a public transport network, it is required to represent a complex network in the form of a graph $G = (\overline{N}, S)$, where S is the sum of arcs in the network that form route sections. A route section is a section of a route between two consecutive nodes connected by a route group. The optimization problem is equivalent to the variational inequality $c(v^*) * (v^* - v) \leq 0$ and will have the following form:

$$S = \sum_{s \in S} \int_0^{v_x} c_s(x) dx \rightarrow min; \tag{9}$$

$$\begin{cases} \sum\limits_{r \in R_w} h_r = T_w \forall w \in W; \\ \sum\limits_{r \in R_w} \delta_{sr} h_r = V_s \forall s \in S; \\ v_i^s = \dfrac{f_l * V_s}{f_s}, \forall l \in B_s; \forall s \in S; \end{cases}$$

$$h_r \geq 0, \forall r \in R.$$

where

W are the starting and ending points in the matrix;

w – elements of the group W, in which $w=(i,j)$, and i – number of routes and j – number of stops;

T_w – the total number of passenger trips in the matrix;

L – a sign for designating a public transport route;

R is a group of routes available for public transport passengers;

R – a sign for designating a public transport route;

R_w is a group of public transport routes;

$\overline{h_r}$ – passenger traffic on route r;

s – section of the transport route, indicating the starting point;

S is a group of hauls on routes accessible to public transport passengers;

c_s – the fare for public transport passengers;

δ_{sr} – the matrix takes the value "1" if the bus passes the stop, and "0" in other cases;

V_s – passenger flow on route s;

v_s^l – passenger traffic on a certain leg of route l;

f_l – movement interval on route l;

f_s – movement interval on route s;

B_s – a group of common routes.

B_s – группа общих маршрутов.

$$c_x = t_s + \left(\frac{a}{f_s}\right) + \beta\left(\frac{v_s+\overline{v_s}}{K_s}\right)^n, \tag{10}$$

where

t_s – the travel time in the vehicle;

f_s – interval on a certain leg of route s;

α,n and β are calibration parameters;

K_s – carrying capacity on a certain stage of the route;

V_s – the total number of routes on leg s;

V_s – passenger traffic on competing routes.

The model assumes that passengers choose from all possible routes connecting any two nodes in the public transport network. The chosen path is reduced to minimizing the total travel time (cost). The total travel time consists of: transportation costs, travel time, waiting time, time to reach the stopping point.

It is assumed that the system has a limited capacity and, therefore, the travel time increases in proportion to the number of passengers. In addition, it is assumed that there is congestion at the stops at the moment when the waiting time of the rolling stock increases, which depends on the number of passengers wishing to use these routes for their trips at the same time.

Once a passenger has entered the vehicle, the travel time depends on the actual level of network congestion (public transport flow of individual vehicles).

The model assumes that between each pair of nodes in the public transport network there is a group of "common routes" that are equally attractive to passengers. Thus, at each stop, passengers consider a group of common routes for their trip and preference is given to the first vehicle that belongs to this group and has free space.

The work was carried out within the framework of the project of the Institute of Mathematics of the National Academy of Sciences of the Kyrgyz Republic.

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IN MEMORY OF PROFESSOR ASYLBEK CHEKEEV

Pankov P.S.

Institute of Mathematics of NAS of KR

A.Chekeev's method on splitting a topological space into infinite number of everywhere dense sets by means of continued fractions is set forth.

Keywords: topological space, everywhere dense set, continued fraction.

А.А.Чекеев сунуш кылган, үзгүлтүксүз бөлчөктөрдүн жардамы менен топологиялык мейкиндикти чексиз санда бардык жерде тыгыз көптүктөргө бөлүүнүн усулу баяндалат.

Урунттуу сөздөр: топологиялык мейкиндик, бардык жерде тыгыз көптүк, үзгүлтүксүз бөлчөк.

Изложен метод А.А.Чекеева расщепления топологического пространства на бесконечное количество всюду плотных множеств с помощью цепных дробей.

Ключевые слова: топологическое пространство, всюду плотное множество, цепная дробь.

Asylbek Asakeevich Chekeev (1962-2022), doctor of physical-mathematical sciences, professor, Corresponding member of National Academy of Sciences of Kyrgyz Republic.

We collaborated. To investigate the task to state recognizability of marked topological spaces put by Academician A.Borubaev, I needed a splitting a topological space into infinite number of everywhere dense sets. A.Chekeev proposed the following construction. Consider all infinite continuous fractions in the segment $[0; 1]$. Let $C(n)$ be the set of all continuous fractions such that all partial quotients since any place are equal to the natural number n.

Obviously, if $n \neq m$ then $C(n) \neq C(m)$ and $C(n)$ is everywhere dense in $[0; 1]$.

We did not have time to publish this result and its consequences during his lifetime.

His memory will always live in our hearts.

CONTENTS

Mathematical logic, algebra and number theory

Mathematical and instrumental methods of economics

Personalia

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9 771694 817007